Models for Retrieval and Browsing

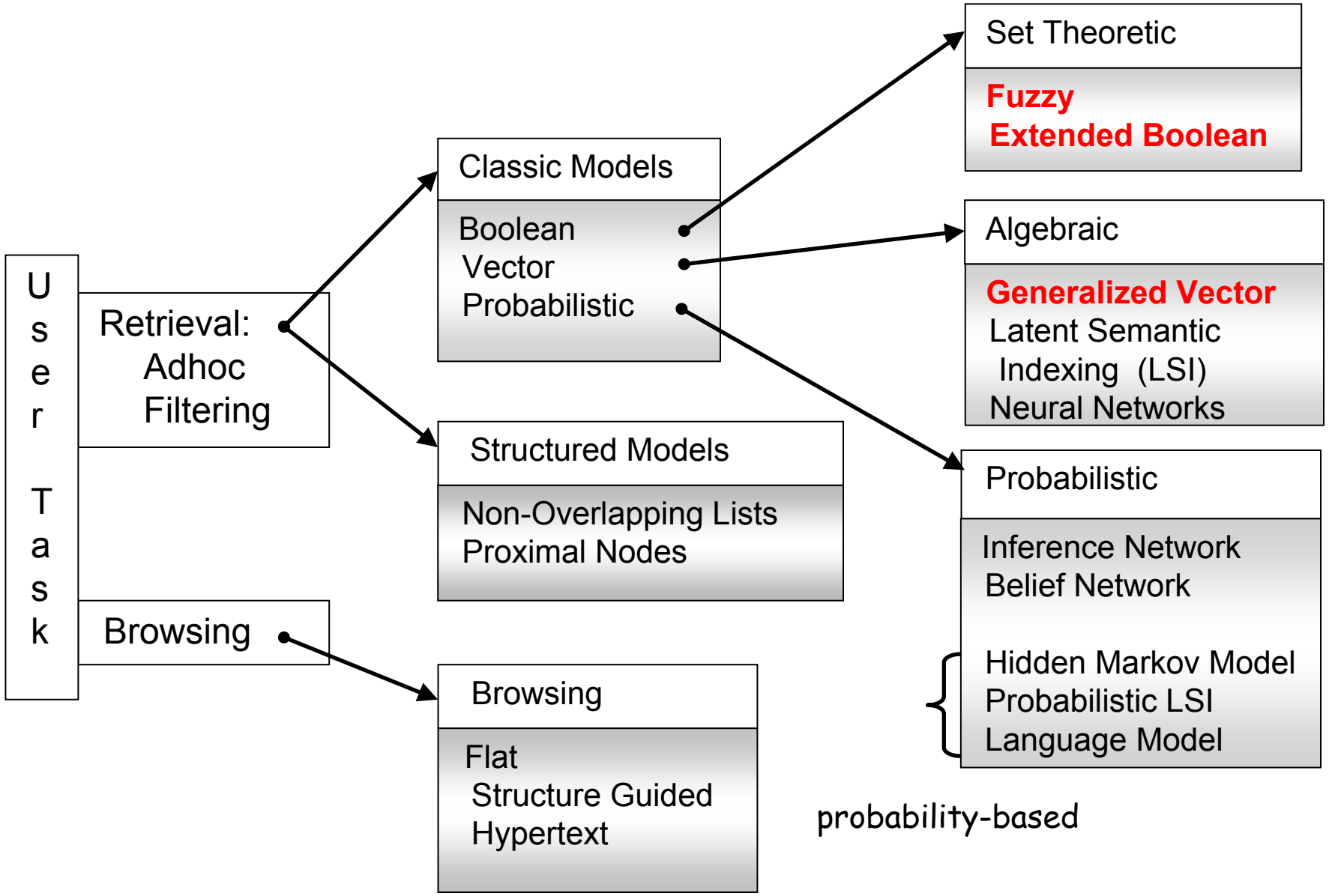
- Fuzzy Set, Extended Boolean, Generalized Vector Space Models

Berlin Chen 2005

Reference:

1. *Modern Information Retrieval*. Chapter 2

Taxonomy of Classic IR Models

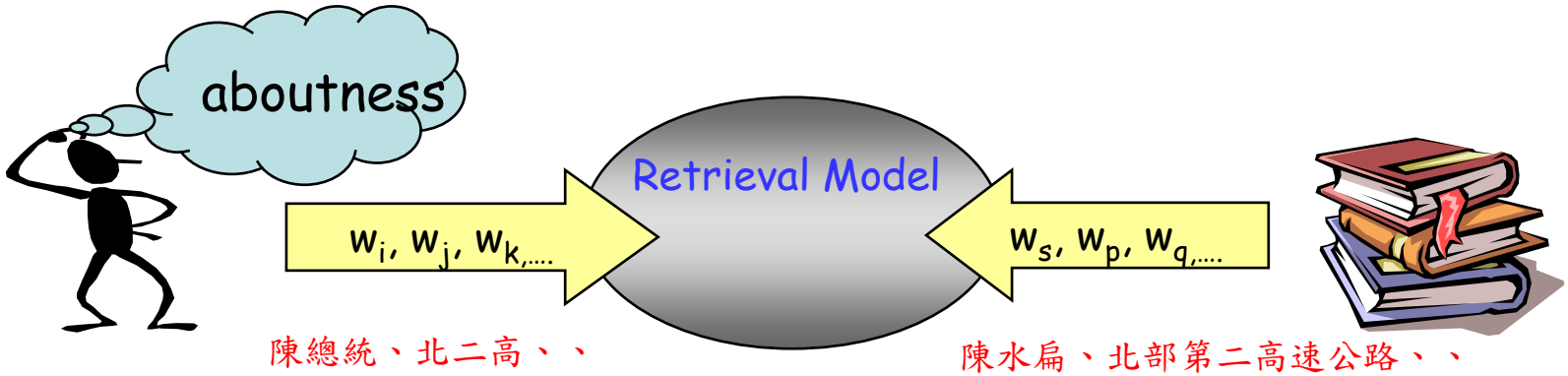


Outline

- Alternative Set Theoretic Models
  - Fuzzy Set Model (Fuzzy Information Retrieval)
  - Extended Boolean Model
- Alternative Algebraic Models
  - Generalized Vector Space Model

Fuzzy Set Model

- Premises
  - Docs and queries are represented through sets of keywords, therefore the matching between them is vague
    - Keywords cannot completely describe the user's information need and the doc's main theme



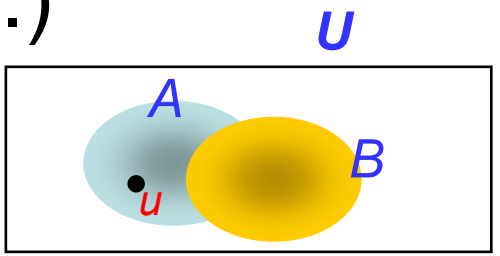
  - For each query term (keyword)
    - Define a fuzzy set and that each doc has a degree of membership (0~1) in the set

Fuzzy Set Model (cont.)

- Fuzzy Set Theory
  - Framework for representing classes (sets) whose boundaries are not well defined
  - Key idea is to introduce the notion of a *degree of membership* associated with the elements of a set
  - This degree of membership varies from 0 to 1 and allows modeling the notion of *marginal* membership
    - 0 $\rightarrow$ no membership
    - 1 $\rightarrow$ full membership
  - Thus, membership is now gradual instead of abrupt
    - Not as conventional Boolean logic

Here we will define a fuzzy set for each query (or index) term, thus each doc has a degree of membership in this set.

Fuzzy Set Model (cont.)



- Definition
  - A fuzzy subset *A* of a universal of discourse *U* is characterized by a membership function
    μ_A: $U \rightarrow [0,1]$
    - Which associates with each element *u* of *U* a number $\mu_A(u)$ in the interval [0,1]
  - Let *A* and *B* be two fuzzy subsets of *U*. Also, let $\overline{A}$ be the complement of *A*. Then,
    - Complement $\mu_{\overline{A}}(u) = 1 - \mu_A(u)$
    - Union $\mu_{A \cup B}(u) = \max(\mu_A(u), \mu_B(u))$
    - Intersection $\mu_{A \cap B}(u) = \min(\mu_A(u), \mu_B(u))$

Fuzzy Set Model (cont.)

- Fuzzy information retrieval
  - Fuzzy sets are modeled based on a **thesaurus**
  - This thesaurus can be constructed by a term-term correlation matrix (or called keyword connection matrix)
    - $\vec{c}$: a term-term correlation matrix
    - $c_{i,l}$: a normalized correlation factor for terms k_i and k_l

Defining term relationship

$$c_{i,l} = \frac{n_{i,l}}{n_i + n_l - n_{i,l}}$$

ranged from 0 to 1

n_i : no of docs that contain k_i

$n_{i,l}$: no of docs that contain both k_i and k_l

docs, paragraphs, sentences, ..

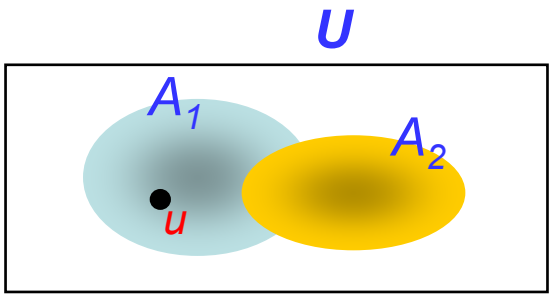
    - We now have the notion of proximity among index terms

  - The relationship is symmetric !

$$\mu_{k_i}(k_l) = c_{i,l} = c_{l,i} = \mu_{k_l}(k_i)$$

Fuzzy Set Model (cont.)

- The union and intersection operations are modified here



$$ab + \bar{a}b + a\bar{b}$$
$$= ab + (1-a)b + a(1-b)$$
$$= ab + b - ab + a - ab$$
$$= 1 - (1 - a - b + ab)$$
$$= 1 - (1-a)(1-b)$$

- Union: algebraic sum (instead of max)

$$\mu_{A_1 \cup A_2}(u) = \mu_{A_1}(u)\mu_{A_2}(u) + \mu_{\bar{A}_1}(u)\mu_{A_2}(u) + \mu_{A_1}(u)\mu_{\bar{A}_2}(u)$$
$$= 1 - \prod_{j=1}^{2}\left(1 - \mu_{A_j}(u)\right)$$

a negative algebraic product

$$\mu_{A_1 \cup A_2 \cdots \cup A_n}(u) = \mu_{\cup_j A_j}(u) = 1 - \prod_{j=1}^{n}\left(1 - \mu_{A_j}(u)\right)$$

- Intersection: algebraic product (instead of min)

$$\mu_{A_1 \cap A_2}(u) = \mu_{A_1}(u)\mu_{A_2}(u)$$

$$\mu_{A_1 \cap A_2 \cdots \cap A_n}(u) = \prod_{j=1}^{n}\mu_{A_j}(u)$$

Fuzzy Set Model (cont.)

– The degree of membership between a doc d_j and an index term k_i

algebraic sum (a doc is a union of index terms)

$$\mu_{k_i}(d_j) = \mu_{d_j}(k_i) = \mu_{\bigcup_{k_l \in d_j} k_l}(k_i)$$

$$= 1 - \prod_{k_l \in d_j} \left(1 - \mu_{k_l}(k_i)\right) = 1 - \prod_{k_l \in d_j} \left(1 - c_{i,l}\right)$$

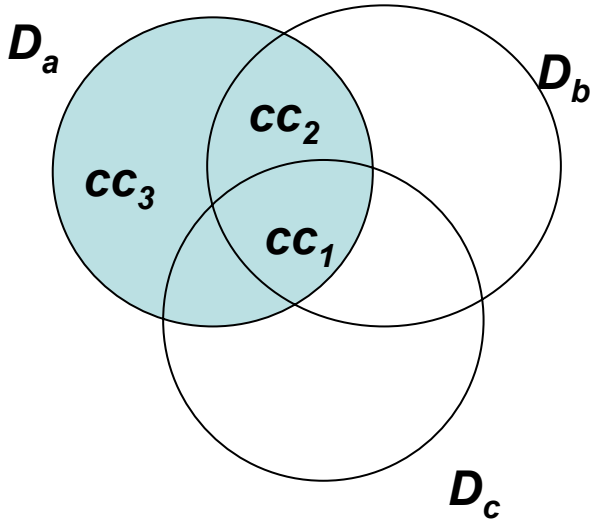
	k_a	k_b
k_i	$c_{i,a}$	$c_{i,b}$
	$1 - c_{i,a}$	$1 - c_{i,b}$

- Computes an **algebraic sum** over all terms in the doc d_j
  - Implemented as the complement of a negative algebraic product
  - A doc d_j belongs to the fuzzy set associated to the term k_i if its own terms are related to k_i
- If there is at least one index term k_l of d_j which is strongly related to the index k_i ($c_{i,l} \sim 1$) then $\mu_{k_i,d_j} \sim 1$
  - k_i is a good fuzzy index for doc d_j
  - And vice versa

Fuzzy Set Model (cont.)

- Example:
  - Query $q=k_a \wedge (k_b \vee \neg k_c)$
    $\vec{q}_{dnf}=(k_a \wedge k_b \wedge k_c) \vee (k_a \wedge k_b \wedge \neg k_c) \vee (k_a \wedge \neg k_b \wedge \neg k_c)$
    $=cc_1+cc_2+cc_3$ ← conjunctive component

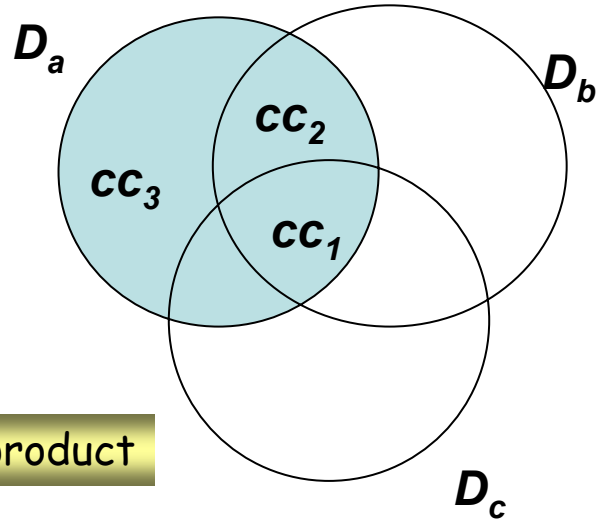
disjunctive normal form



  - D_a is the fuzzy set of docs associated to the term k_a
  - Degree of membership ?

Fuzzy Set Model (cont.)

- Degree of membership



algebraic sum

$$\mu_{q,d_j} = \mu_{cc_1 \cup cc_2 \cup cc_3, d_j}$$

for a doc d_j in the fuzzy answer set D_q

negative algebraic product

$$= 1 - \prod_{i=1}^{3} (1 - \mu_{cc_i, d_j})$$

$$= 1 - \left(1 - \mu_{\overbrace{a \cap b \cap c}^{cc_1}, d_j}\right)\left(1 - \mu_{\overbrace{a \cap b \cap \bar{c}}^{cc_2}, d_j}\right)\left(1 - \mu_{\overbrace{a \cap \bar{b} \cap \bar{c}}^{cc_3}, d_j}\right)$$

algebraic product

$$= 1 - (1 - \mu_{a,d_j}\mu_{b,d_j}\mu_{c,d_j})$$

$$\times (1 - \mu_{a,d_j}\mu_{b,d_j}(1 - \mu_{c,d_j})) \times (1 - \mu_{a,d_j}(1 - \mu_{b,d_j})(1 - \mu_{c,d_j}))$$

Fuzzy Set Model (cont.)

- Advantages
  - The correlations among index terms are considered
  - Degree of relevance between queries and docs can be achieved
- Disadvantages
  - Fuzzy IR models have been discussed mainly in the literature associated with fuzzy theory
  - Experiments with standard test collections are not available

Extended Boolean Model

Salton et al., 1983

- Motive
  - Extend the Boolean model with the functionality of partial matching and term weighting 陳水扁 及 呂秀蓮
    - E.g.: in Boolean model, for the qery $q=k_x \wedge k_y$, a doc contains either k_x or k_y is as irrelevant as another doc which contains neither of them
    - How about the disjunctive query $q=k_x \vee k_y$ 陳水扁 或 呂秀蓮
  - Combine Boolean query formulations with characteristics of the vector model
    - Term weighting
    - Algebraic distances for similarity measures

a ranking can be obtained

Extended Boolean Model (cont.)

- Term weighting
  - The weight for the term k_x in a doc d_j is

$$w_{x,j} = tf_{x,j} \times \frac{idf_x}{\max_i idf_i}$$

normalized frequency

Normalized *idf*

ranged from 0 to 1

    - $w_{x,j}$ is normalized to lay between 0 and 1

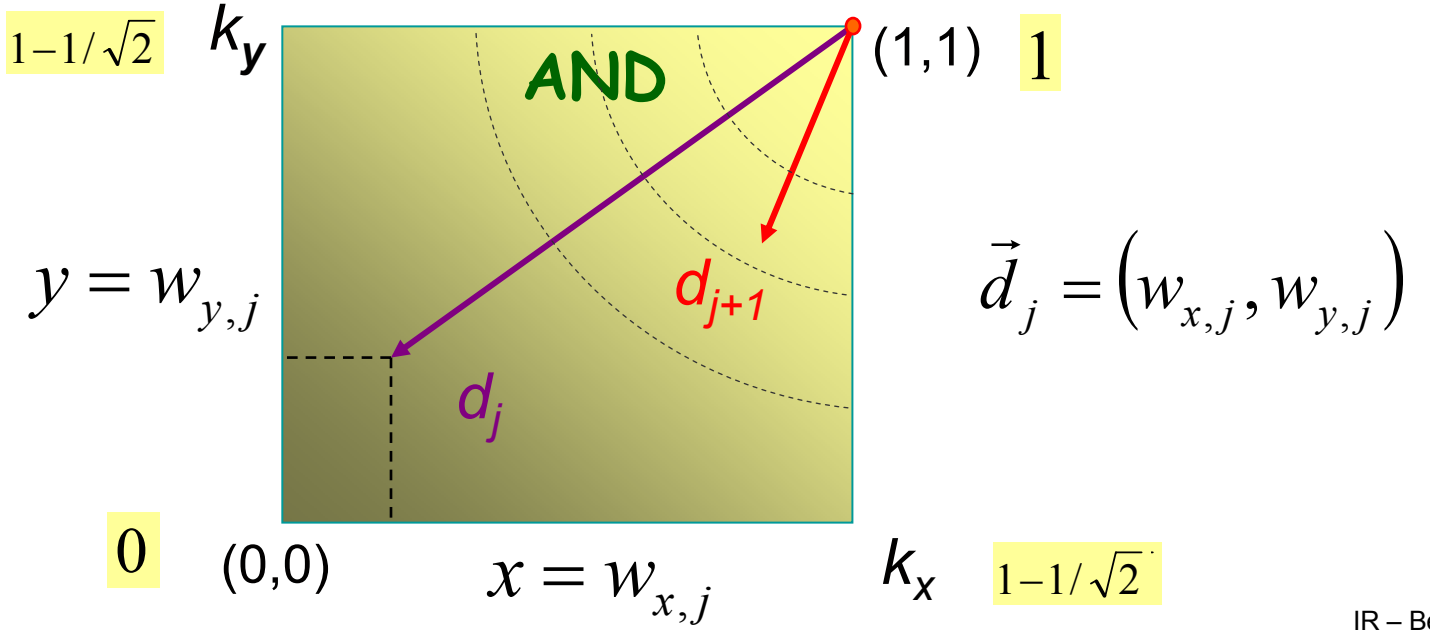
- Assume two index terms k_x and k_y were used
  - Let x denote the weight $w_{x,j}$ of term k_x on doc d_j
  - Let y denote the weight $w_{y,j}$ of term k_y on doc d_j
  - The doc vector $\vec{d}_j = (w_{x,j}, w_{y,j})$ is represented as $d_j = (x, y)$
  - Queries and docs can be plotted in a two-dimensional map

Extended Boolean Model (cont.)

- If the query is $q=k_x \wedge k_y$ (conjunctive query)
  - -The docs near the point (1,1) are preferred
  - -The similarity measure is defined as

$$sim(q_{and}, d) = 1 - \sqrt{\frac{(1-x)^2 + (1-y)^2}{2}}$$

2-norm model
(Euclidean distance)

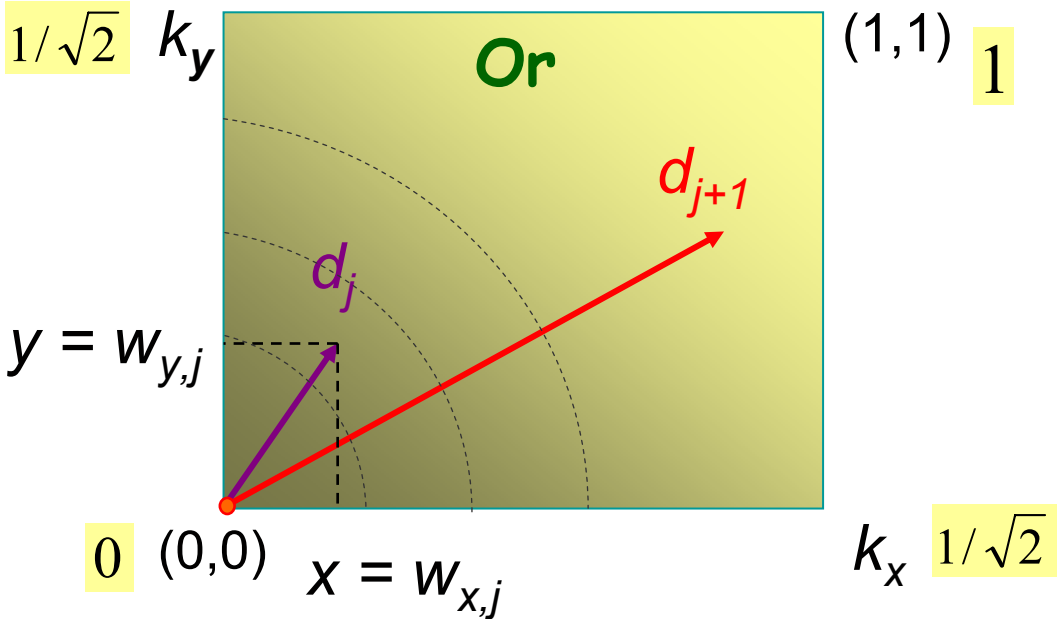


Extended Boolean Model (cont.)

- If the query is $q=k_x \vee k_y$ (disjunctive query)
  - -The docs far from the point (0,0) are preferred
  - -The similarity measure is defined as

$$sim\ (q_{or}, d) = \sqrt{\frac{x^2 + y^2}{2}}$$

2-norm model
(Euclidean distance)



Extended Boolean Model (cont.)

- The similarity measures $sim\left(q_{or}, d\right)$ and $sim\left(q_{and}, d\right)$ also lay between 0 and 1

Extended Boolean Model (cont.)

- Generalization
  - *t* index terms are used → *t*-dimensional space
  - *p*-norm model, $1 \le p \le \infty$

$$q_{and} = k_1 \wedge^p k_2 \wedge^p \ldots \wedge^p k_m \Rightarrow sim(q_{and}, d) = 1 - \left(\frac{(1-x_1)^p + (1-x_2)^p + \ldots + (1-x_m)^p}{m} \right)^{\frac{1}{p}}$$

$$q_{or} = k_1 \vee^p k_2 \vee^p \ldots v^p k_m \Rightarrow sim(q_{or}, d) = \left(\frac{x_1^{\,p} + x_2^{\,p} + \ldots + x_m^{\,p}}{m} \right)^{\frac{1}{p}}$$

  - Some interesting properties
    - *p=1* ⇒ $sim(q_{and}, d) = sim(q_{or}, d) = \frac{x_1 + x_2 + \ldots + x_m}{m}$
    - *p*= ∞ ⇒ $sim(q_{and}, d) \approx \min(x_i)$, $sim(q_{or}, d) \approx \max(x_i)$ — just like the formula of fuzzy logic

Extended Boolean Model (cont.)

- Example query 1: $q = (k_1 \wedge^p k_2) \vee^p k_3$
  - Processed by grouping the operators in a predefined order

$$sim(q, d) = \left(\frac{\left(1 - \left(\frac{(1 - x_1)^p + (1 - x_2)^p}{2} \right)^{\frac{1}{p}} \right)^p + x_3^p}{2} \right)^{\frac{1}{p}}$$

- Example query 2: $q = (k_1 \vee^2 k_2) \wedge^\infty k_3$
  - Combination of different algebraic distances

$$sim(q, d) = \min\left(\left(\frac{x_1^2 + x_2^2}{2} \right)^{\frac{1}{2}}, x_3 \right)$$

Extended Boolean Model (cont.)

- Advantages
  - A hybrid model including properties of both the set theoretic models and the algebraic models
    - Relax the Boolean algebra by interpreting Boolean operations in terms of algebraic distances
- Disadvantages
  - Distributive operation does not hold for ranking computation
    - E.g.: $q_1 = (k_1 \wedge^2 k_2) \vee^2 k_3, q_2 = (k_1 \vee^2 k_3) \wedge^2 (k_2 \vee^2 k_3)$

$$\left(\frac{\left(1-\left(\frac{(1-x_1)^2+(1-x_2)^2}{2}\right)^{\frac{1}{2}}\right)^2+x_3^2}{2}\right)^{\frac{1}{2}} \qquad sim\ (q_1, d) \neq sim\ (q_2, d) \qquad 1-\left(\frac{\left(1-\left(\frac{x_1^2+x_2^2}{2}\right)\right)^2+\left(1-\left(\frac{x_2^2+x_3^2}{2}\right)\right)^2}{2}\right)^{\frac{1}{2}}$$

  - Assumes mutual independence of index terms

Generalized Vector Model

Wong et al., 1985

- Premise
  - Classic models enforce independence of index terms
  - For the **Vector model**
    - Set of term vectors $\{\vec{k_1}, \vec{k_1}, ..., \vec{k_t}\}$ are linearly independent and form a basis for the subspace of interest
    - Frequently, it means pairwise orthogonality

      $\forall i,j \Rightarrow \vec{k_i} \bullet \vec{k_j} = \vec{0}$ (in a more restrictive sense)

- Wong et al. proposed an interpretation
  - The index term vectors are linearly independent, but not pairwise orthogonal
    - Generalized Vector Model

Generalized Vector Model (cont.)

- **Key idea**
  - Index term vectors form the basis of the space are not orthogonal and are represented in terms of smaller components (**minterms**)

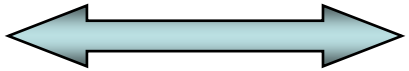
- **Notations**
  - $\{k_1, k_2, ..., k_t\}$: the set of all terms
  - $w_{i,j}$: the weight associated with $[k_i, d_j]$
  - **Minterms**:binary indicators (0 or 1) of all patterns of occurrence of terms within documents
    - Each represent one kind of co-occurrence of index terms in a specific document

Generalized Vector Model (cont.)

- Representations of **minterms**

m_1=(0,0,….,0)
m_2=(1,0,….,0)
m_3=(0,1,….,0)
m_4=(1,1,….,0)
m_5=(0,0,1,..,0)
…
m_{2^t}=(1,1,1,..,1)

2^t minterms



$\overrightarrow{m_1}$=(1,0,0,0,0,….,0)
$\overrightarrow{m_2}$=(0,1,0,0,0,….,0)
$\overrightarrow{m_3}$=(0,0,1,0,0,….,0)
$\overrightarrow{m_4}$=(0,0,0,1,0,….,0)
$\overrightarrow{m_5}$=(0,0,0,0,1,….,0)
…
$\overrightarrow{m_{2^t}}$=(0,0,0,0,0,….,1)

2^t minterm vectors

Points to the docs where only index terms k_1 and k_2 co-occur and the other index terms disappear

Point to the docs containing all the index terms

Pairwise orthogonal vectors $\overrightarrow{m_i}$ associated with minterms m_i as the **basis** for the **generalized vector space**

Generalized Vector Model (cont.)

- Minterm vectors are **pairwise orthogonal**. But, this does not mean that the index terms are independent
  - Each **minterm** specifies a kind of dependence among index terms
  - That is, the co-occurrence of index terms inside docs in the collection induces dependencies among these index terms

Generalized Vector Model (cont.)

- The vector associated with the term k_i is represented by **summing** up all minterms containing it and **normalizing**

$$\vec{k}_i = \frac{\sum_{\forall r, g_i(m_r)=1} c_{i,r}\,\vec{m}_r}{\sqrt{\sum_{\forall r, g_i(m_r)=1} c_{i,r}^{\,2}}}$$

- The weight associated with the pair $[k_i, m_r]$ sums up the weights of the term k_i in all the docs which have a term occurrence pattern given by m_r.
- Notice that for a collection of size N, only N minterms affect the ranking (and not 2^N)

$$c_{i,r} = \sum_{d_j \,|\, g_l(\vec{d}_j)=g_l(m_r), \text{for all } l} w_{i,j}$$

All the docs whose term co-occurrence relation (pattern) can be represented as (exactly coincide with that of) minterm m_r

$g_i(m_r)$ Indicates the index term k_i is in the minterm m_r

Generalized Vector Model (cont.)

- The similarity between the query and doc is calculated in the space of minterm vectors

$$\vec{d}_j = \sum_i w_{i,j}\vec{k}_i \quad \Rightarrow \quad = \sum_r s_{j,r}\vec{m}_r$$

$$\vec{q}_j = \sum_i w_{i,q}\vec{k}_i \quad \Rightarrow \quad = \sum_r s_{q,r}\vec{m}_r$$

t-dimensional

2^t-dimensional

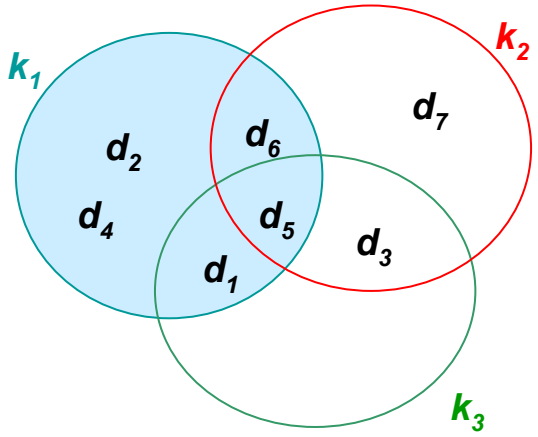
$$sim\left(\vec{q}_j, \vec{d}_j\right) = \frac{\sum_i w_{i,q} \cdot w_{i,j}}{\sqrt{\sum_i w_{i,q}}\sqrt{\sum_i w_{i,q}}}$$

$$sim\left(\vec{q}_j, \vec{d}_j\right) = \frac{\sum_r s_{q,r} \cdot s_{d,r}}{\sqrt{\sum_r s_{q,r}}\sqrt{\sum_r s_{d,r}}}$$

Generalized Vector Model (cont.)

- **Example** (a system with three index terms)

minterm	k_1	k_2	k_3
m_1	0	0	0
m_2	1	0	0
m_3	0	1	0
m_4	1	1	0
m_5	0	0	1
m_6	1	0	1
m_7	0	1	1
m_8	1	1	1



$$\vec{k}_1 = \frac{c_{1,2}\vec{m}_2 + c_{1,4}\vec{m}_4 + c_{1,6}\vec{m}_6 + c_{1,8}\vec{m}_8}{\sqrt{c_{1,2}^{\ 2} + c_{1,4}^{\ 2} + c_{1,6}^{\ 2} + c_{1,8}^{\ 2}}}$$

$$\vec{k}_2 = \frac{c_{2,3}\vec{m}_3 + c_{2,4}\vec{m}_4 + c_{2,7}\vec{m}_7 + c_{2,8}\vec{m}_8}{\sqrt{c_{2,3}^{\ 2} + c_{2,4}^{\ 2} + c_{2,7}^{\ 2} + c_{2,8}^{\ 2}}}$$

$$\vec{k}_3 = \frac{c_{3,5}\vec{m}_5 + c_{3,6}\vec{m}_6 + c_{3,7}\vec{m}_7 + c_{3,8}\vec{m}_8}{\sqrt{c_{3,5}^{\ 2} + c_{3,6}^{\ 2} + c_{3,7}^{\ 2} + c_{3,8}^{\ 2}}}$$

	k_1	k_2	k_3	minterm
d_1	2	0	1	m_6
d_2	1	0	0	m_2
d_3	0	1	3	m_7
d_4	2	0	0	m_2
d_5	1	2	4	m_8
d_6	1	2	0	m_4
d_7	0	5	0	m_3
q	1	2	3	

$$c_{1,2} = w_{1.2} + w_{1.4} = 1 + 2 = 3 \qquad \vec{k}_1 = \frac{3\vec{m}_2 + 1\vec{m}_4 + 2\vec{m}_6 + 1\vec{m}_8}{\sqrt{3^2 + 1^2 + 2^2 + 1^2}}$$

$$c_{1,4} = w_{1.6} = 1$$

$$c_{1,6} = w_{1,1} = 2$$

$$c_{1,8} = w_{1,5} = 1$$

$$c_{2,3} = w_{2,7} = 5$$

$$c_{2,4} = w_{2,6} = 2$$

$$c_{2,7} = w_{2,3} = 1$$

$$c_{2,8} = w_{2,5} = 2$$

$$\vec{k}_2 = \frac{5\vec{m}_3 + 2\vec{m}_4 + 1\vec{m}_7 + 2\vec{m}_8}{\sqrt{5^2 + 2^2 + 1^2 + 2^2}}$$

$$c_{3,5} = 0$$

$$c_{3,6} = w_{3,1} = 1$$

$$c_{3,7} = w_{3,3} = 3$$

$$c_{3,8} = w_{3,5} = 4$$

$$\vec{k}_3 = \frac{0\vec{m}_5 + 1\vec{m}_6 + 3\vec{m}_7 + 4\vec{m}_8}{\sqrt{0^2 + 1^2 + 3^2 + 4^2}}$$

Generalized Vector Model (cont.)

- **Example**: Ranking

$$\vec{k}_1 = \frac{3\vec{m}_2 + 1\vec{m}_4 + 2\vec{m}_6 + 1\vec{m}_8}{\sqrt{3^2+1^2+2^2+1^2}} = \frac{3\vec{m}_2 + 1\vec{m}_4 + 2\vec{m}_6 + 1\vec{m}_8}{\sqrt{15}}$$

$$\vec{k}_2 = \frac{5\vec{m}_3 + 2\vec{m}_4 + 1\vec{m}_7 + 2\vec{m}_8}{\sqrt{5^2+2^2+1^2+2^2}} = \frac{5\vec{m}_3 + 2\vec{m}_4 + 1\vec{m}_7 + 2\vec{m}_8}{\sqrt{34}}$$

$$\vec{k}_3 = \frac{0\vec{m}_5 + 1\vec{m}_6 + 3\vec{m}_7 + 4\vec{m}_8}{\sqrt{0^2+1^2+3^2+4^2}} = \frac{1\vec{m}_6 + 3\vec{m}_7 + 4\vec{m}_8}{\sqrt{26}}$$

$$\vec{d}_1 = 2\vec{k}_1 + 1\vec{k}_3$$

$$= \frac{2\cdot 3}{\sqrt{15}}\vec{m}_2 + \frac{2\cdot 1}{\sqrt{15}}\vec{m}_4 + \left(\frac{2\cdot 2}{\sqrt{15}} + \frac{1\cdot 1}{\sqrt{26}}\right)\vec{m}_6 + \frac{1\cdot 3}{\sqrt{26}}\vec{m}_7 + \left(\frac{2\cdot 1}{\sqrt{15}} + \frac{1\cdot 4}{\sqrt{26}}\right)\vec{m}_8$$

($s_{d_1,2}$, $s_{d_1,4}$, $s_{d_1,6}$, $s_{d_1,7}$, $s_{d_1,8}$)

$$\vec{q} = 1\vec{k}_1 + 2\vec{k}_2 + 3\vec{k}_3$$

$$= \frac{1\cdot 3}{\sqrt{15}}\vec{m}_2 + \frac{2\cdot 5}{\sqrt{34}}\vec{m}_3 + \left(\frac{1\cdot 1}{\sqrt{15}} + \frac{2\cdot 2}{\sqrt{34}}\right)\vec{m}_4 + \left(\frac{1\cdot 2}{\sqrt{15}} + \frac{3\cdot 1}{\sqrt{26}}\right)\vec{m}_6 + \left(\frac{2\cdot 1}{\sqrt{34}} + \frac{3\cdot 3}{\sqrt{26}}\right)\vec{m}_7 + \left(\frac{1\cdot 1}{\sqrt{15}} + \frac{2\cdot 2}{\sqrt{34}} + \frac{3\cdot 4}{\sqrt{26}}\right)\vec{m}_8$$

($s_{q,2}$, $s_{q,3}$, $s_{q,4}$, $s_{q,6}$, $s_{q,7}$, $s_{q,8}$)

$$sim(q,d) = \text{consine}(q,d) = \frac{\sum_{r|s_{q,r}\neq 0 \wedge s_{d,r}\neq 0} s_{q,r}\cdot s_{d,r}}{\sqrt{\sum_{r|s_{q,r}\neq 0 \wedge s_{d,r}\neq 0} s_{q,r}^2}\sqrt{\sum_{r|s_{q,r}\neq 0 \wedge s_{d,r}\neq 0} s_{d,r}^2}}$$

The similarity between the query and doc is calculated in the space of minterm vectors

$$sim(q,d_1) = \frac{s_{q,2}s_{d_1,2} + s_{q,4}s_{d_1,4} + s_{q,6}s_{d_1,6} + s_{q,7}s_{d_1,7} + s_{q,8}s_{d_1,8}}{\sqrt{s_{q,2}^2 + s_{q,3}^2 + s_{q,4}^2 + s_{q,6}^2 + s_{q,7}^2 + s_{q,8}^2}\sqrt{s_{d_1,2}^2 + s_{d_1,4}^2 + s_{d_1,6}^2 + s_{d_1,7}^2 + s_{d_1,8}^2}}$$

Generalized Vector Model (cont.)

- Term Correlation
  - The degree of correlation between the terms k_i and k_j can now be computed as

$$\vec{k}_i \bullet \vec{k}_j = \sum_{\forall r | g_i(m_r)=1 \wedge g_j(m_r)=1} c_{i,r} \times c_{j,r}$$

    - Do not need to be normalized? (because we have done it before!)

Generalized Vector Model (cont.)

- Advantages
  - Model considers correlations among index terms
  - Model does introduce interesting new ideas

- Disadvantages
  - Not clear in which situations it is superior to the standard vector model
  - Computation costs are higher