Informed Search and Exploration

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Reference:

1. S. Russell and P. Norvig. Artificial Intelligence: A Modern Approach, Chapter 4
2. S. Russell's teaching materials

Introduction

- Informed Search
  - Also called heuristic search
  - Use problem-specific knowledge
  - Search strategy: a node is selected for exploration based on an evaluation function, $f(n)$
    - Estimate of desirability

- Evaluation function generally consists of two parts
  - The path cost from the initial state to a node *n*, $g(n)$ (optional)
  - The estimated cost of the cheapest path from a node *n* to a goal node, the heuristic function, $h(n)$
    - If the node *n* is a goal state $\rightarrow$ $h(n)=0$
    - Can't be computed from the problem definition (need experience)

Heuristics

- Used to describe rules of thumb or advise that are generally effective, but not guaranteed to work in every case

- In the context of search, a heuristic is a function that takes a state as an argument and returns a number that is an estimate of the merit of the state with respect to the goal

- Not all heuristic functions are beneficial
  - Should consider the time spent on evaluating the heuristic function
  - Useful heuristics should be computationally inexpensive

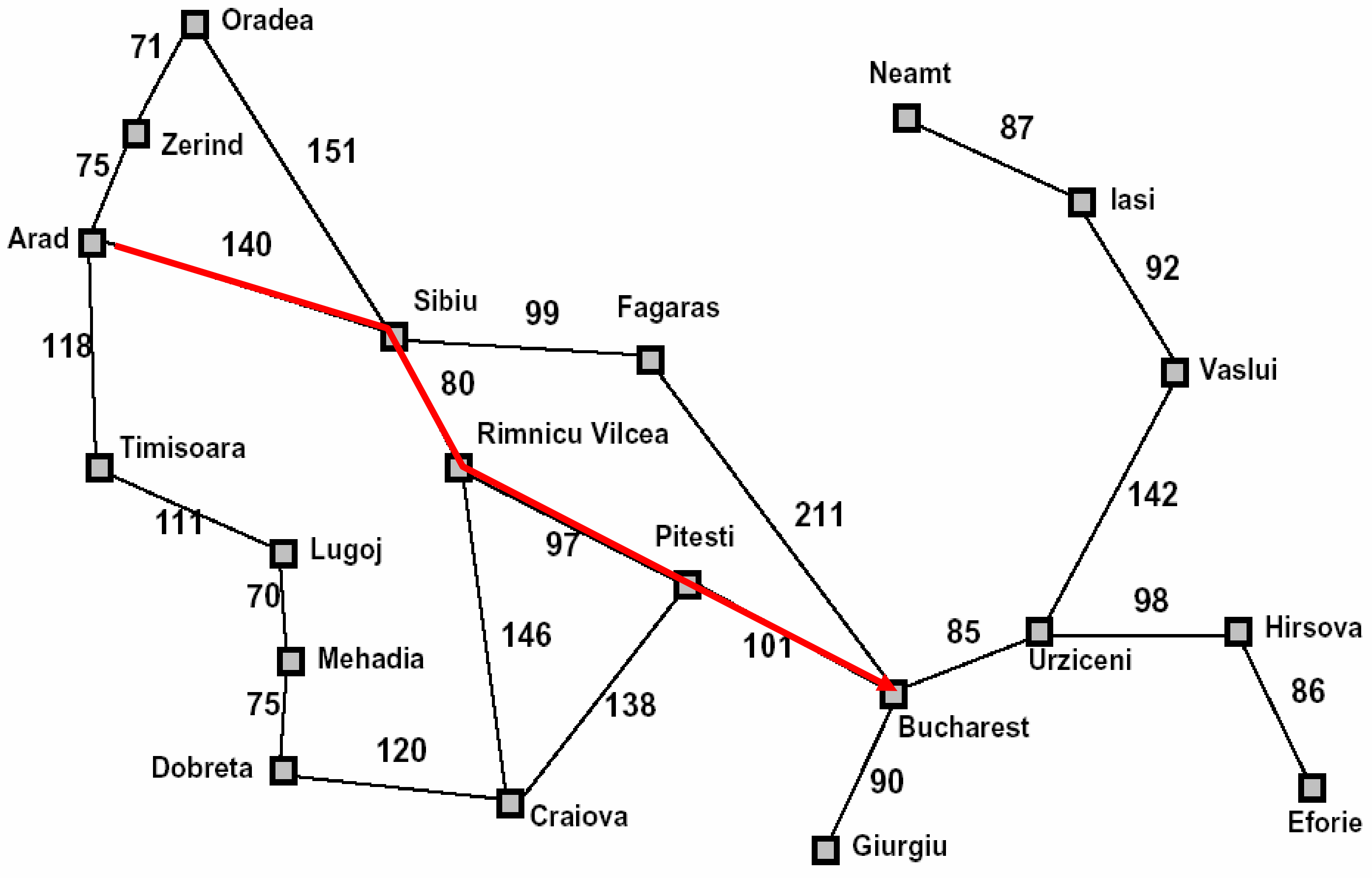
Best-First Search

- Choose the most desirable (seemly-best) node for expansion based on evaluation function
  - Lowest cost/highest probability evaluation
- Implementation
  - *Fringe* is a priority queue in decreasing order of desirability

- Several kinds of best-first search introduced
  - **Greedy best-first search**
  - **A* search**
  - Iterative-Deepening A* search
  - Recursive best-first search
  - Simplified memory-bounded A* search

memory-bounded
heuristic search

Map of Romania



$h(n)$

Straight−line distance
to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
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Rimnicu Vilcea	193
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Greedy Best-First Search

- Expand the node that appears to be closet to the goal, based on the heuristic function only

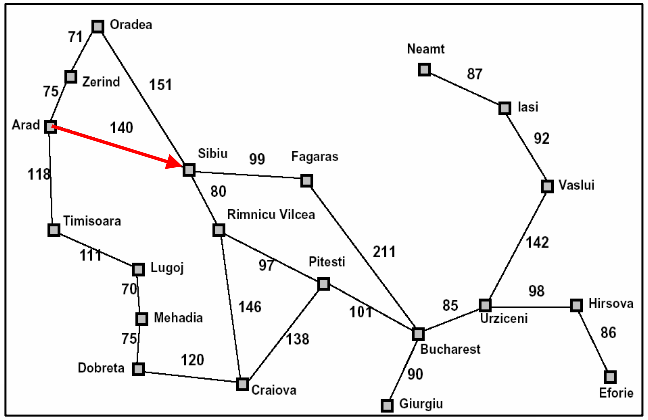
$$f(n) = h(n) = \text{estimate of cost from node } n \text{ to the closest goal}$$

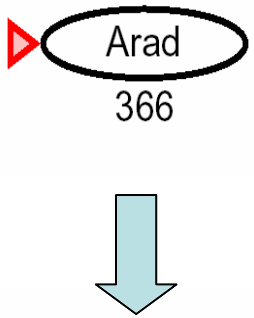
  – E.g., the straight-line distance heuristics h_{SLD} to Bucharest for the route-finding problem
    - $h_{SLD}(In(Arad)) = 366$

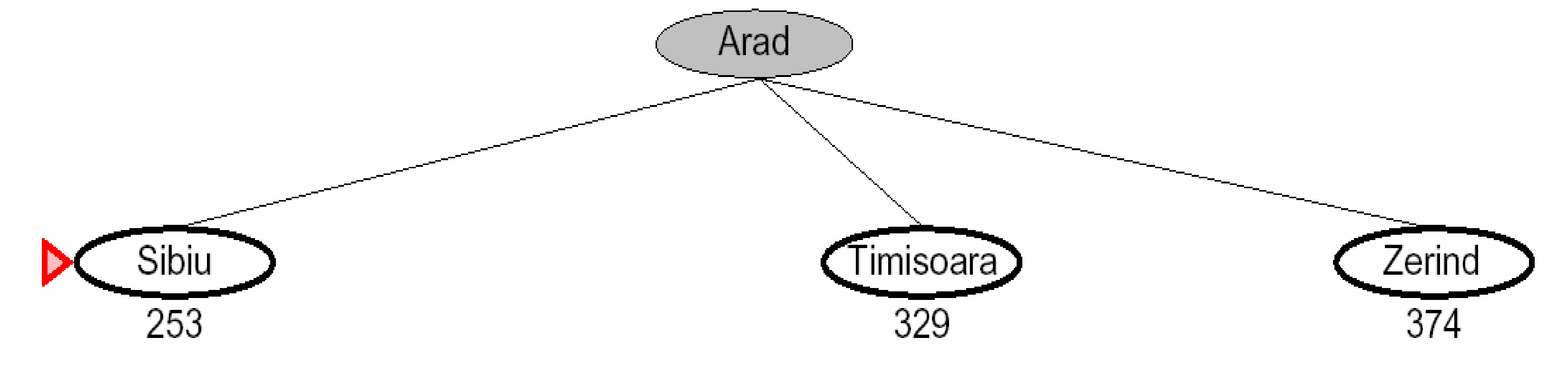
- "greedy" – at each search step the algorithm always tries to get close to the goal as it can

Greedy Best-First Search (cont.)

- Example 1: the route-finding problem

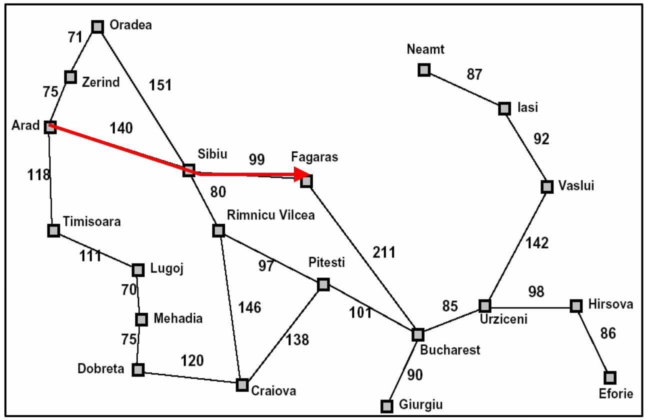


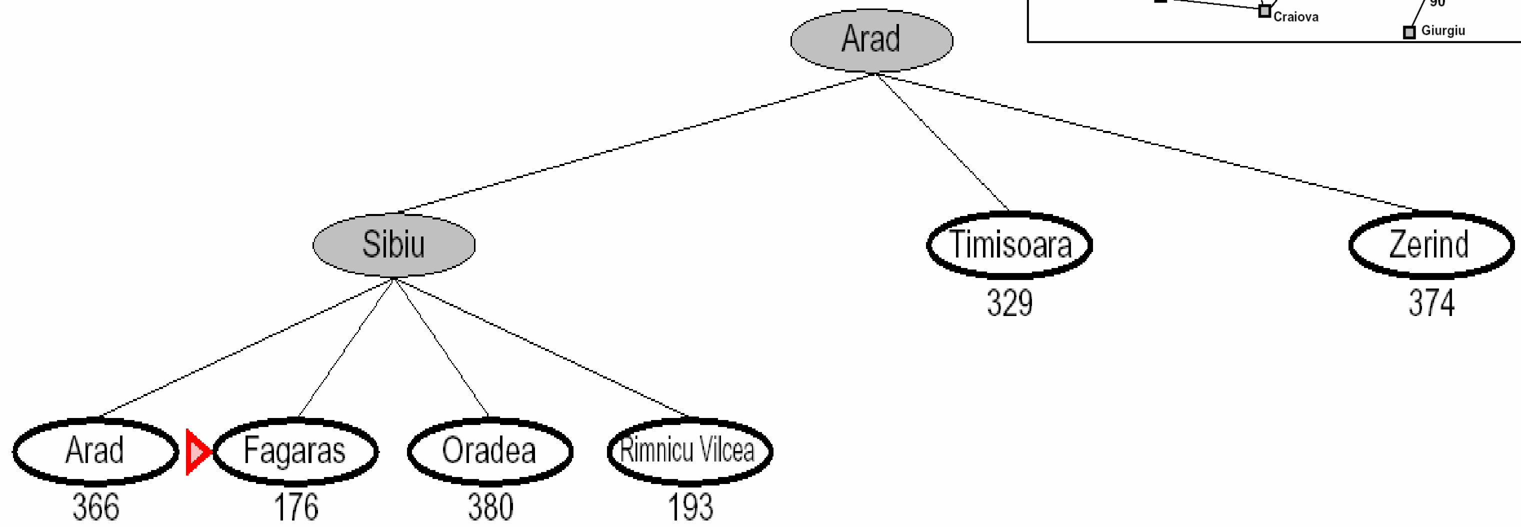




Greedy Best-First Search (cont.)

- Example 1: the route-finding problem





Greedy Best-First Search (cont.)

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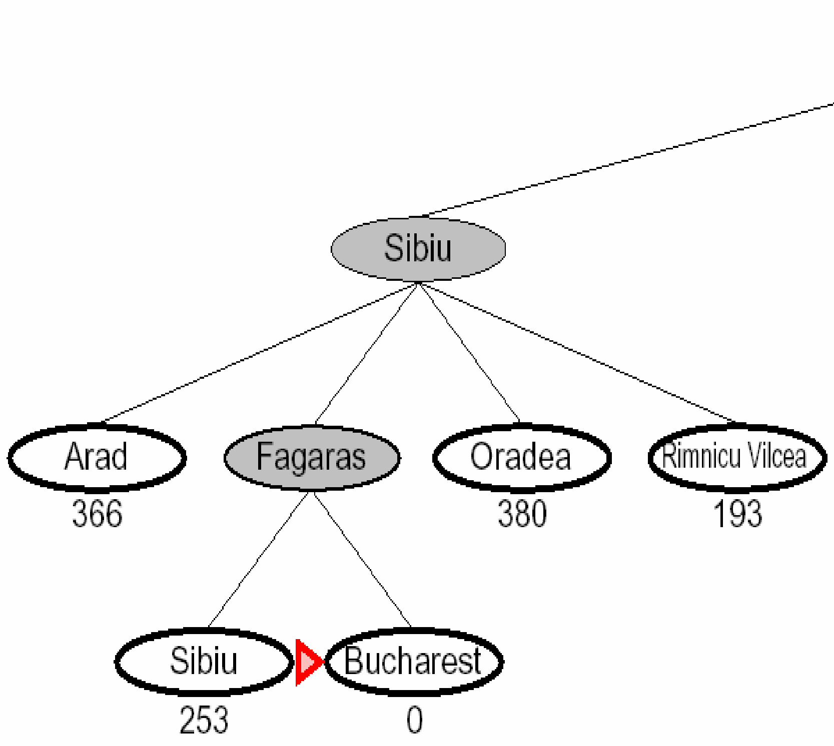
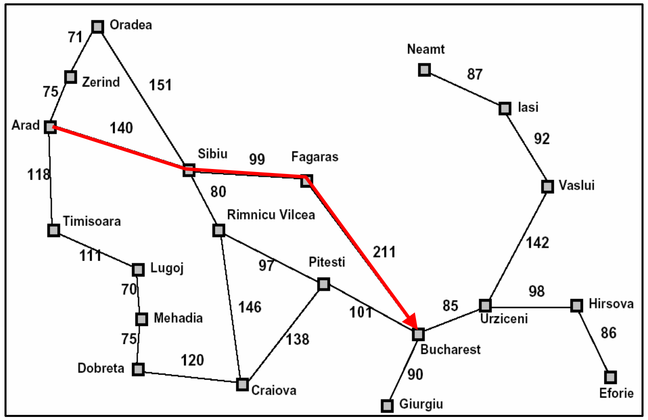


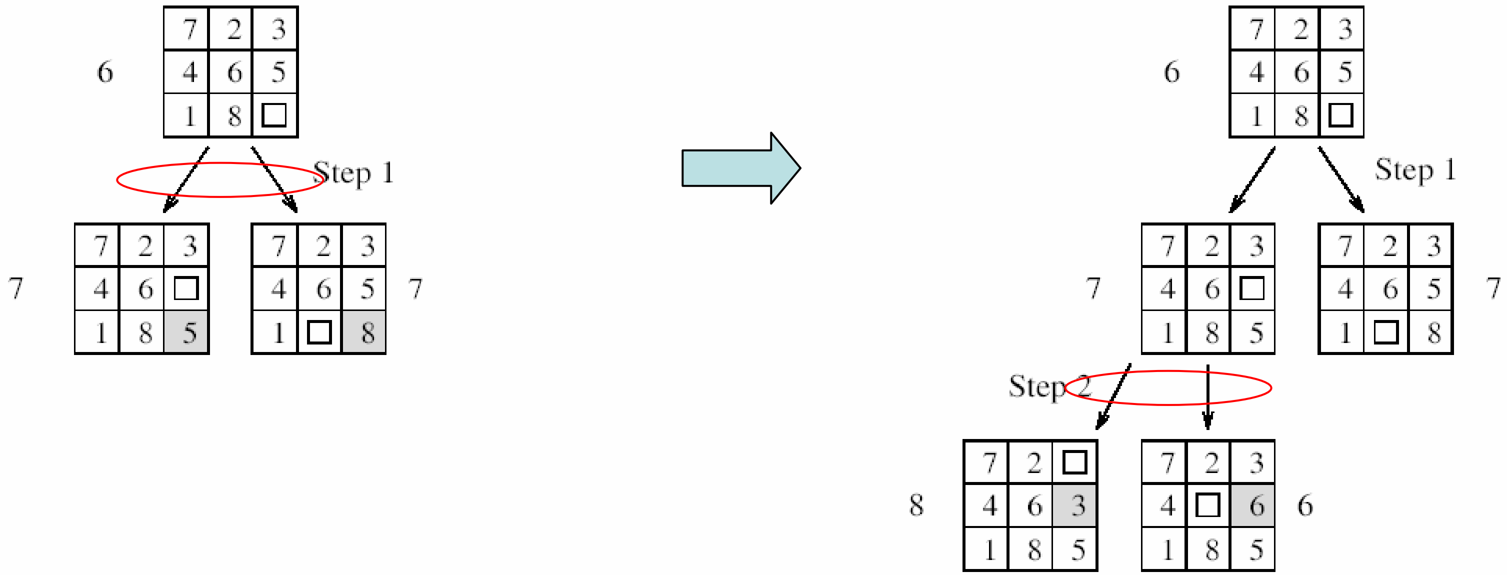


  - The solution is not optimal (?)

Greedy Best-First Search (cont.)

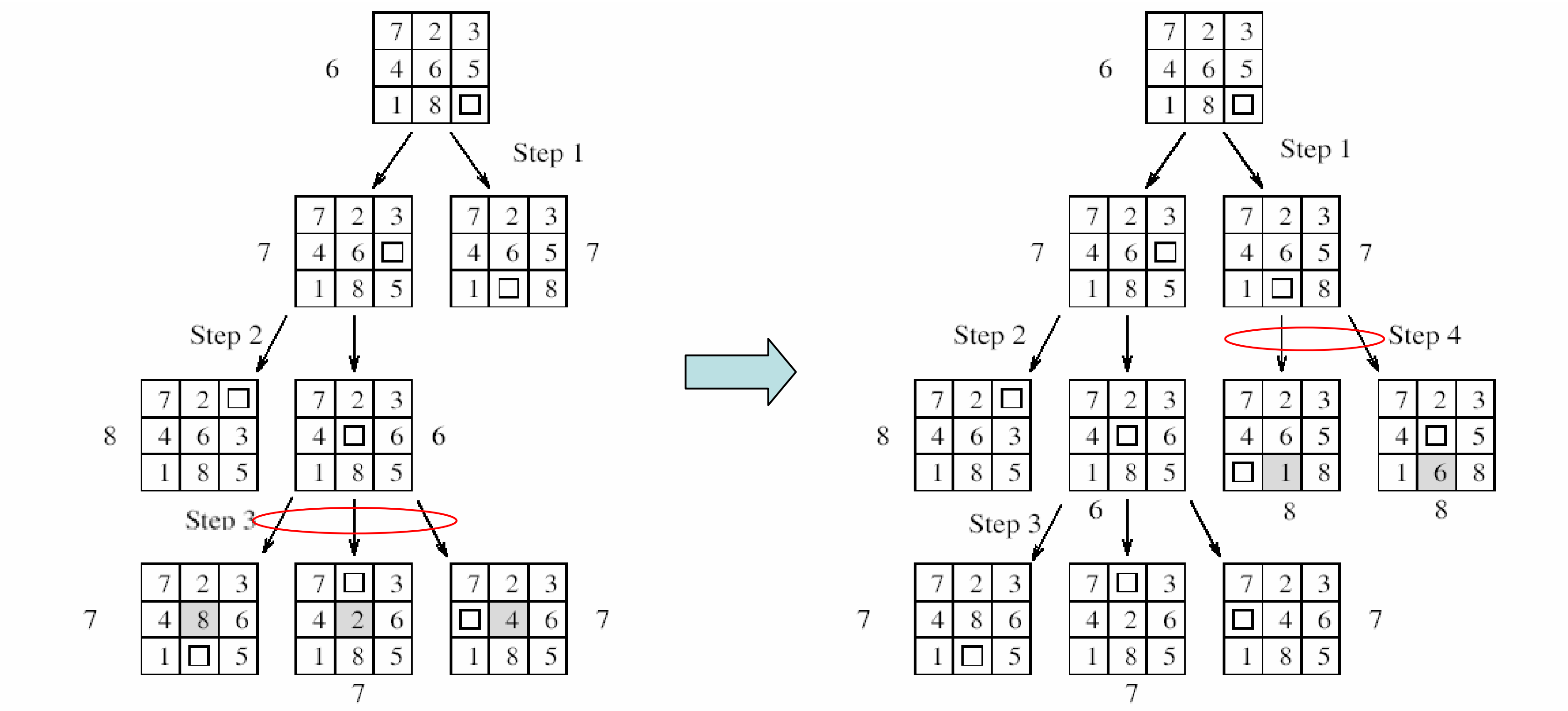
- Example 2: the 8-puzzle problem

2+0+0+0+1+1+2+0=6 (Manhattan distance)



Greedy Best-First Search (cont.)

- Example 2: the 8-puzzle problem (cont.)



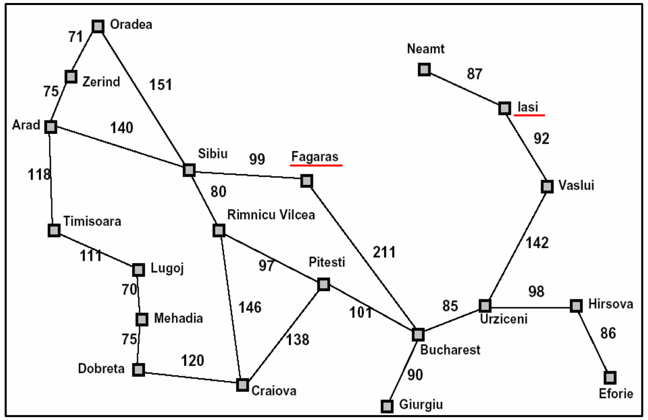
(c)

Figure 11.6 Applying best-first search to the 8-puzzle: (a) initial configuration; (b) final configuration; and (c) states resulting from the first four steps of best-first search. Each state is labeled with its h-value (that is, the Manhattan distance from the state to the final state).

Properties of Greedy Best-First Search

- Prefer to follow a single path all the way to the goal, and will back up when dead end is hit (like DFS)
  - Also have the possibility to go down infinitely

- Is neither optimal nor complete
  - Not complete: could get suck in loops
    - E.g., finding path from Iasi to Fagars



- Time and space complexity
  - Worse case: $O(b^m)$
  - But a good heuristic function could give dramatic improvement

A* Search

Hart, Nilsson, Raphael, 1968

- Pronounced as "A-star search"
- Expand a node by evaluating the path cost to reach itself, $g(n)$, and the estimated path cost from it to the goal, $h(n)$
  - Evaluation function

$$f(n) = g(n) + h(n)$$

$$g(n) = \text{path cost so far to reach } n$$
$$h(n) = \text{estimated path cost to goal from } n$$
$$f(n) = \text{estimated total path cost through } n \text{ to goal}$$

  - Uniform-cost search + greedy best-first search ?
  - Avoid expanding nodes that are already expansive

A* Search (cont.)

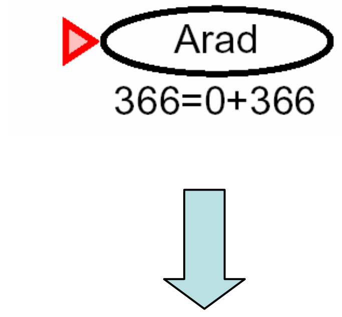
- A* is optimal if the heuristic function $h(n)$ never overestimates
  - Or say "if the heuristic function is admissible"
  - E.g. the straight-line-distance heuristics are admissible

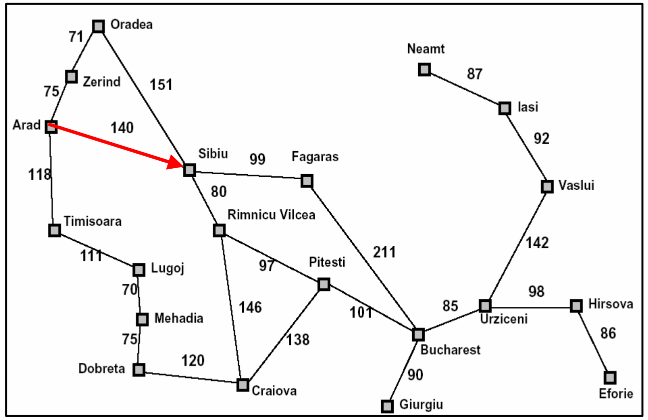
$$\boxed{\begin{array}{l} h(n) \le h^*(n), \\ \text{where } h^*(n) \text{ is the true path cost from } n \text{ to goal} \end{array}}$$

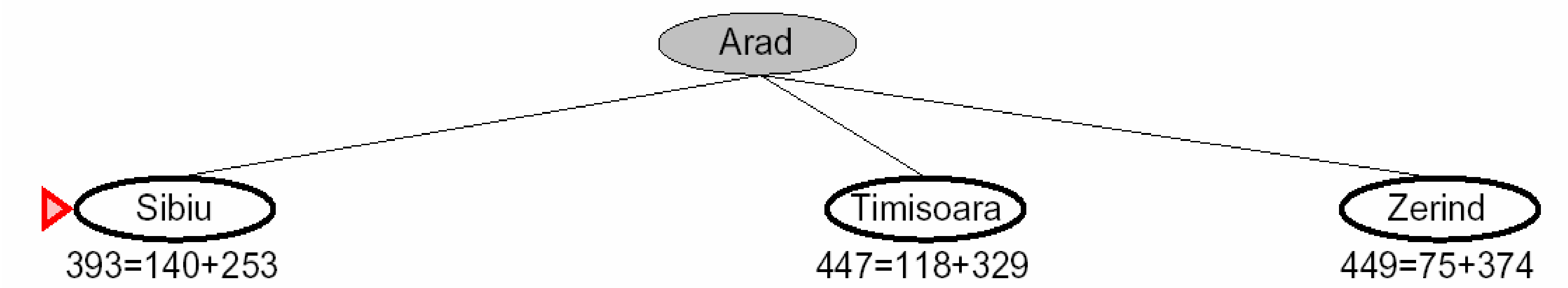
Finding the shortest-path goal

A* Search (cont.)

- Example 1: the route-finding problem







A* Search (cont.)

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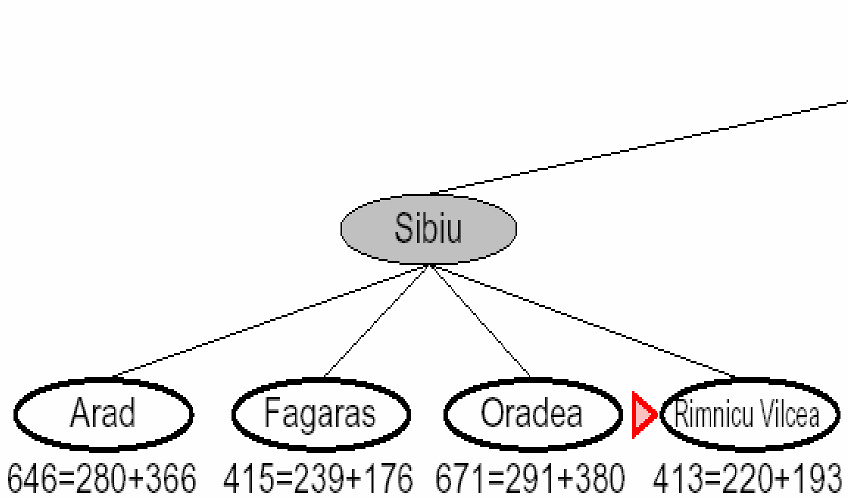
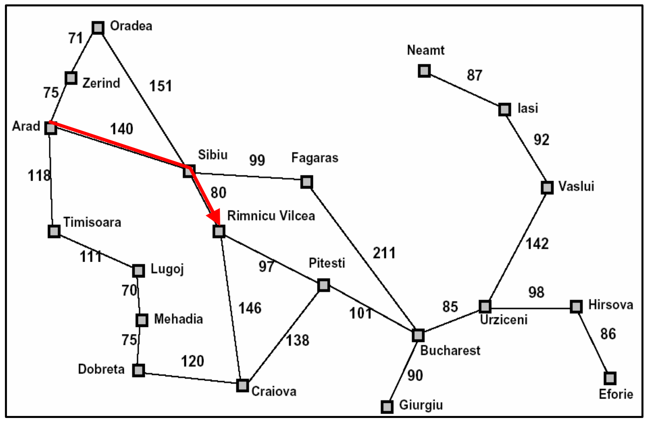
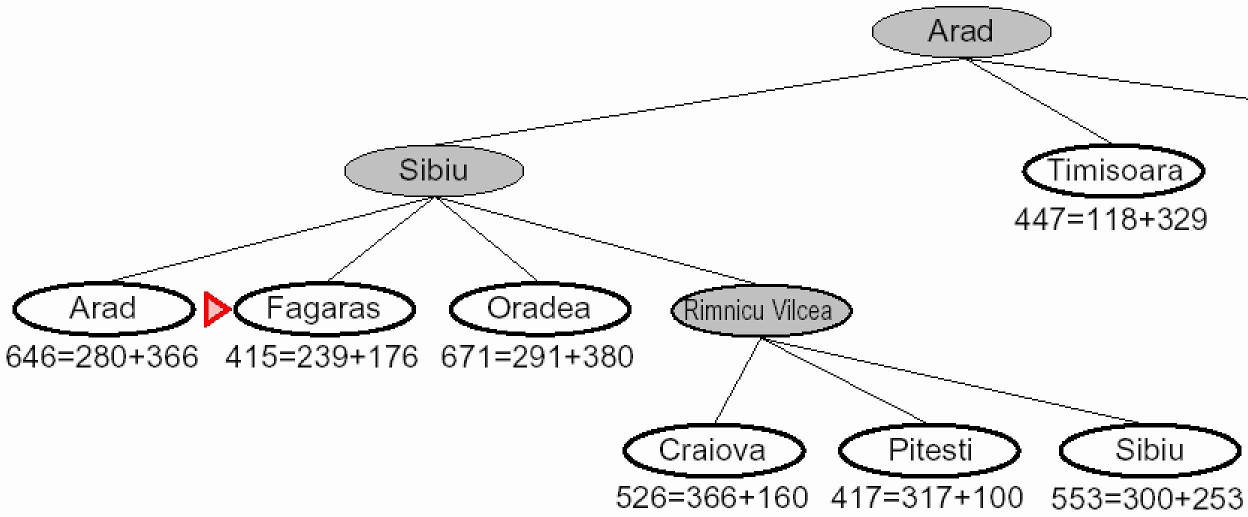
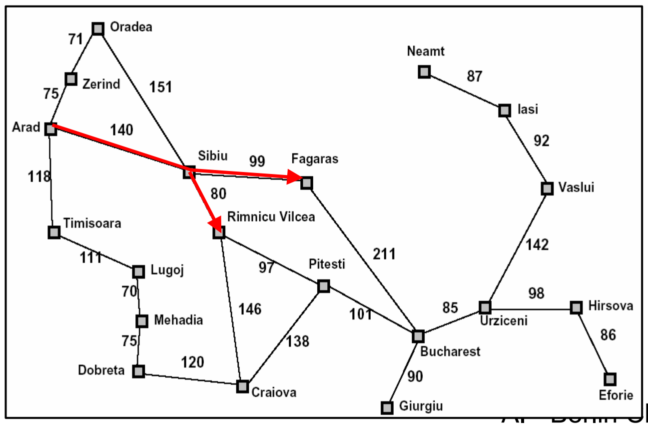




A* Search (cont.)

- Example 1: the route-finding problem





A* Search (cont.)

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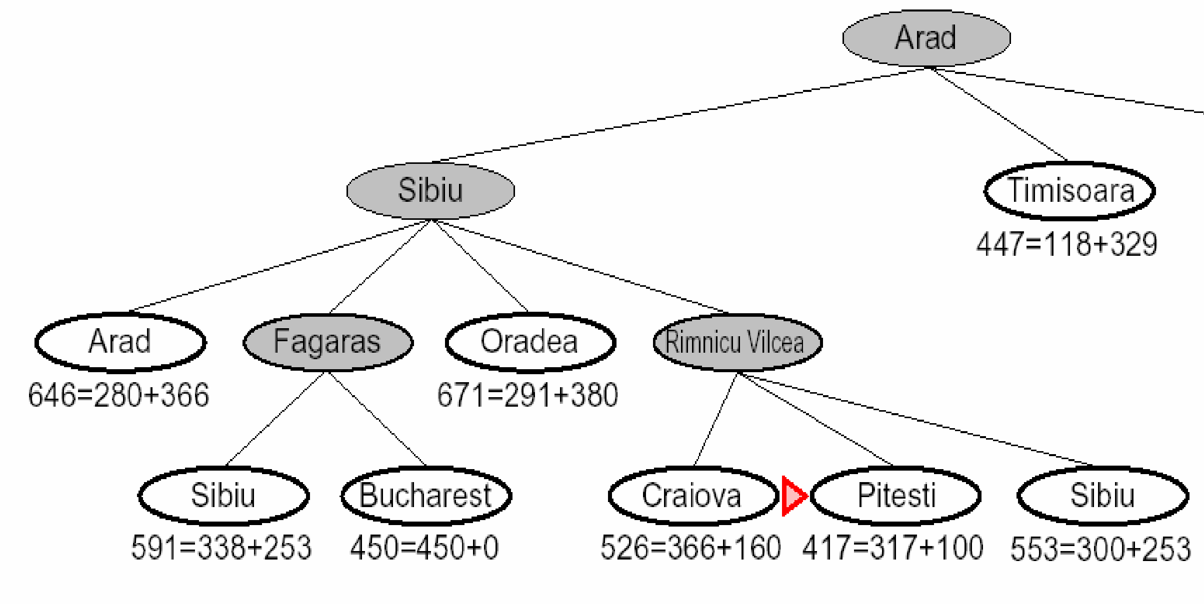

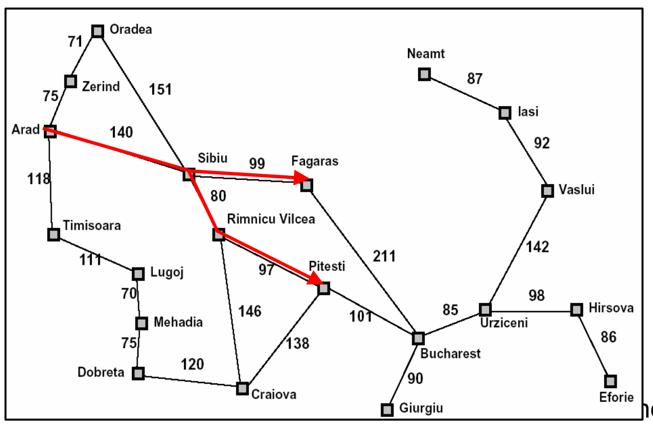

A* Search (cont.)

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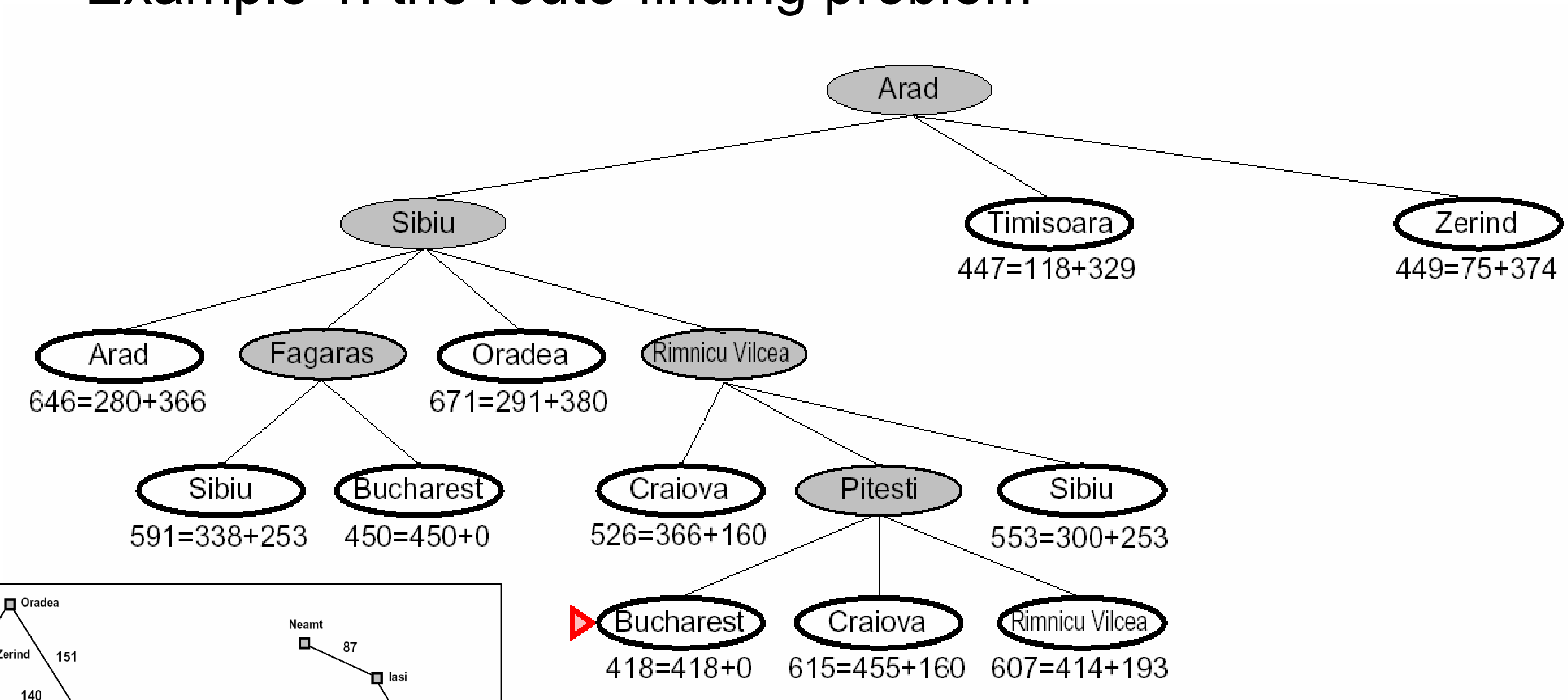
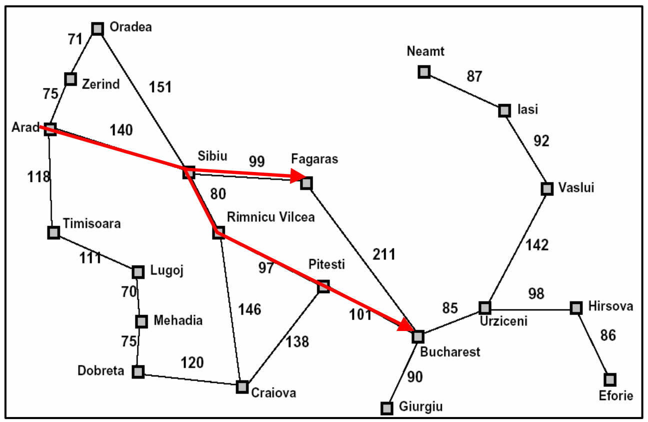


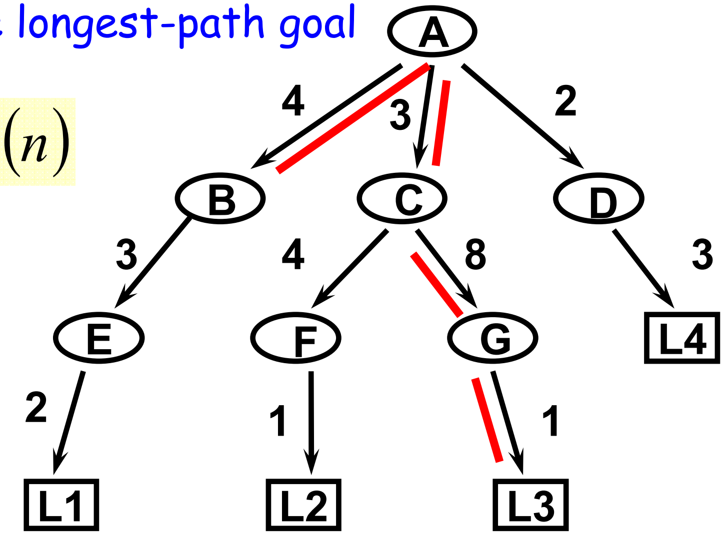


A* Search (cont.)

- Example 2: the state-space just represented as a tree

Finding the longest-path goal

$$h(n) \geq h^*(n)$$



Evaluation function of node n:

$$f(n) = g(n) + h(n)$$

Node	g(n)	h(n)	f(n)
A	0	15	15
B	4	9	13
C	3	12	15
D	2	5	7
E	7	4	11
F	7	2	9
G	11	3	14
L1	9	0	9
L2	8	0	8
L3	12	0	12
L4	5	0	5

Fringe *(sorted)*

Fringe Top	*Fringe Elements*
A(15)	A(15)
C(15)	C(15), B(13), D(7)
G(14)	G(14), B(13), F(9), D(7)
B(13)	B(13), L3(12), F(9), D(7)
L3(12)	L3(12), E(11), F(9), D(7)

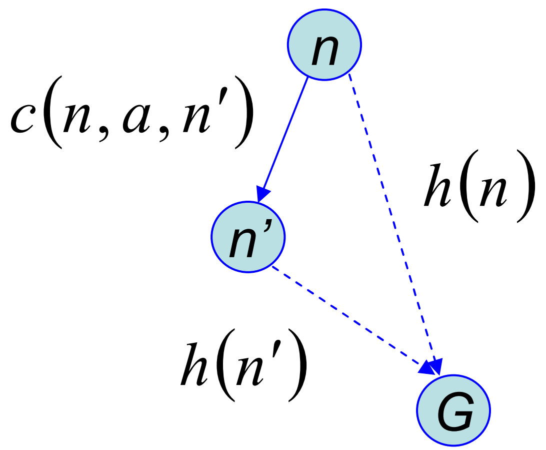
Consistency of A* Heuristics

- A heuristic h is *consistent* if

$$h(n) \leq c(n,a,n') + h(n')$$

successor of n

  – A stricter requirement on h



- If h is consistent (monotonic)

$$\begin{aligned} f(n') &= g(n') + h(n') \\ &= g(n) + c(n,a,n') + h(n') \\ &\geq g(n) + h(n) \\ &\geq f(n) \end{aligned}$$

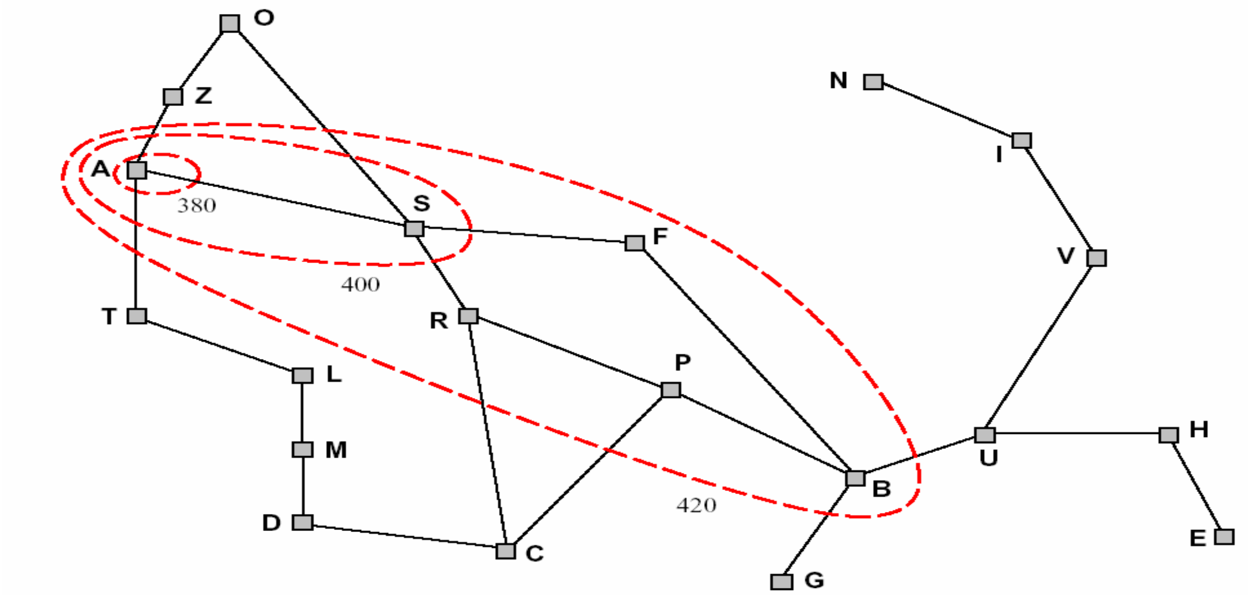
Finding the shortest-path goal

, where $h(\cdot)$ is the straight-line distance to the nearest goal

  – I.e., $f(n)$ is nondecreasing along any path during search

Contours of the Evaluation Functions

- Fringe (leaf) nodes expanded in concentric contours



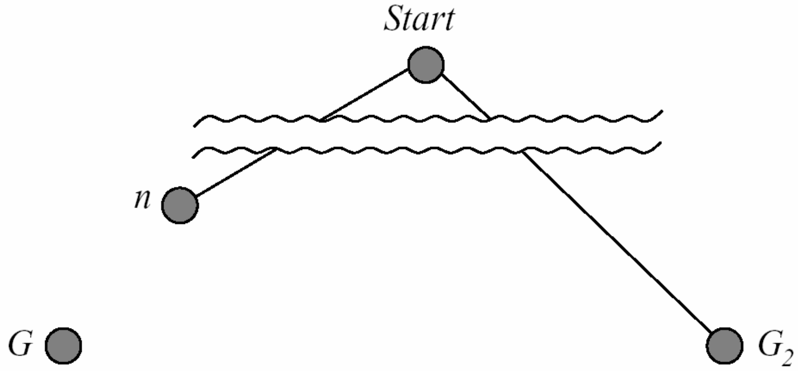
- Uniformed search ($\forall n, h(n) = 0$)
  - Bands circulate around the initial state
- A* search
  - Bands stretch toward the goal and is narrowly focused around the optimal path if more accurate heuristics were used

Contours of the Evaluation Functions (cont.)

- If G is the optimal goal
  - A* search expands all nodes with $f(n) < f(G)$
  - A* search expands some nodes with $f(n)=f(G)$
  - A* expands no nodes with $f(n) > f(G)$

Optimality of A* Search

- A* search is optimal
- Proof
  - Suppose some suboptimal goal G_2 has been generated and is in the fringe (queue)
  - Let *n* be an unexpanded node on a shortest path to an optimal goal *G*



$$
\begin{aligned}
f(G_2) &= g(G_2) && \text{since } h(G_2) = 0 \\
&> g(G)\ (= g(n) + h^*(n)) && \text{since } G_2 \text{ is suboptiomal} \\
&\geq f(n)\ (= g(n) + h(n)) && \text{since } h \text{ is admissible } (h(n) \leq h^*(n))
\end{aligned}
$$

  - A* will never select G_2 for expansion since $f(G_2) > f(n)$

Optimality of A* Search (cont.)

- Another proof
  - Suppose when algorithm terminates, G_2 is a complete path (a solution) on the top of the fringe and a node n that stands for a partial path presents somewhere on the fringe. There exists a complete path G passing through n, which is not equal to G_2 and is optimal (with the lowest path cost)

> 1. G is a complete which passes through node n, $f(G)>=f(n)$
> 2. Because G_2 is on the top of the fringe ,
> $f(G_2)<=f(n)<=f(G)$
> 3. Therefore, it makes contrariety !!

- A* search optimally efficient
  - For any given heuristic function, no other optimal algorithms is guaranteed to expand fewer nodes than A*

Completeness of A* Search

- A* search is complete
  - If every node has a finite branching factor
  - If there are finitely many nodes with $f(n) \leq f(G)$
    - Every infinite path has an infinite path cost

> Proof:
> Because A* adds bands (expands nodes) in order of increasing f, it must eventually reach a band where f is equal to the path to a goal state.

- To Summarize again *If G* is the optimal goal

$$\text{A}^* \text{ expands all nodes with } f(n) < f(G)$$

$$\text{A}^* \text{ expands smoe nodes with } f(n) = f(G)$$

$$\text{A}^* \text{ expands no nodes with } f(n) > f(G)$$

Complexity of A* Search

- Time complexity: $O(b^d)$
- Space complexity: $O(b^d)$
  - Keep all nodes in memory
  - Not practical for many large-scale problems
- Theorem
  - The search space of A* grows exponentially unless the error in the heuristic function grows no faster than the logarithm of the actual path cost

$$\left|h(n) - h^*(n)\right| \leq O\left(\log h^*(n)\right)$$

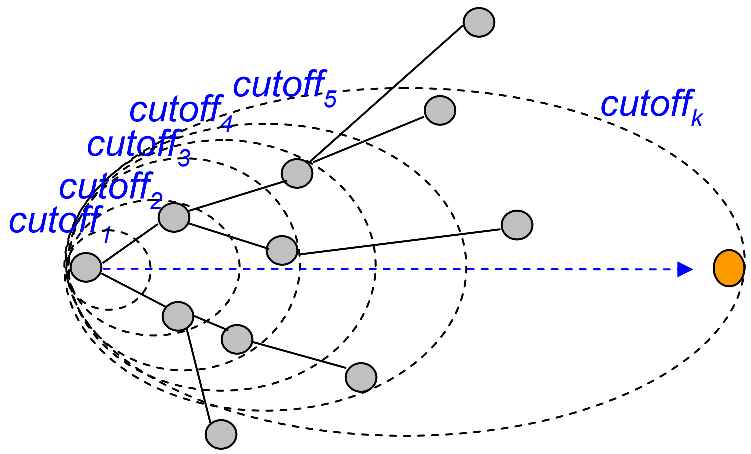
Memory-bounded Heuristic Search

- Iterative-Deepening A* search
- Recursive best-first search
- Simplified memory-bounded A* search

Iterative Deepening A* Search (IDA*)

- The idea of iterative deepening was adapted to the heuristic search context to reduce memory requirements

- At each iteration, DFS is performed by using the f-cost ($g + h$) as the cutoff rather than the depth
  - E.g., the smallest f-cost of any node that exceeded the cutoff on the previous iteration

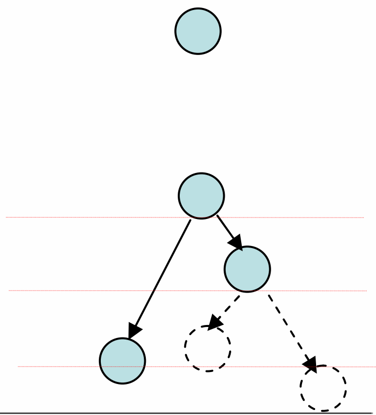


Iterative Deepening A* Search (cont.)

```
function IDA*(problem) returns a solution sequence
   inputs: problem, a problem
   static: f-limit, the current f- COST limit
           root, a node

   root ← MAKE-NODE(INITIAL-STATE[problem])
   f-limit ← f- COST(root)
   loop do
      solution, f-limit ← DFS-CONTOUR(root, f-limit)
      if solution is non-null then return solution
      if f-limit = ∞ then return failure; end
```

Iterations



```
function DFS-CONTOUR(node, f-limit) returns a solution sequence and a new f- COST limit
   inputs: node, a node
           f-limit, the current f- COST limit
   static: next-f, the f- COST limit for the next contour, initially ∞

   if f- COST[node] > f-limit then return null, f- COST[node]
   if GOAL-TEST[problem](STATE[node]) then return node, f-limit
   for each node s in SUCCESSORS(node) do
      solution, new-f ← DFS-CONTOUR(s, f-limit)
      if solution is non-null then return solution, f-limit
      next-f ← MIN(next-f, new-f); end
   return null, next-f
```

Properties of IDA*

- IDA* is complete and optimal
- Space complexity: $O(bf(G)/\delta) \approx O(bd)$
  - δ : the smallest step cost
  - $f(G)$: the optimal solution cost
- Time complexity: $O(\alpha b^d)$
  - α : the number of distinct f values smaller than the optimal goal
- Between iterations, IDA* retains only a single number – the f -cost
- IDA* has difficulties in implementation when dealing with real-valued cost

Recursive Best-First Search (RBFS)

- Attempt to mimic best-first search but use only linear space
  - Can be implemented as a recursive algorithm
  - Keep track of the f -value of the best alternative path from any ancestor of the current node
  - If the current node exceeds the limit, then the recursion unwinds back to the alternative path
  - As the recursion unwinds, the f -value of each node along the path is replaced with the best f-value of its children

Recursive Best-First Search (cont.)

- Algorithm

```
function RECURSIVE-BEST-FIRST-SEARCH(problem) returns a solution, or failure
    RBFS(problem, MAKE-NODE(INITIAL-STATE[problem]), ∞)

function RBFS(problem, node, f_limit) returns a solution, or failure and a new f-cost limit
    if GOAL-TEST[problem](state) then return node
    successors ← EXPAND(node, problem)
    if successors is empty then return failure, ∞
    for each s in successors do
        f[s] ← max(g(s) + h(s), f[node])
    repeat
        best ← the lowest f-value node in successors
        if f[best] > f_limit then return failure, f[best]
        alternative ← the second-lowest f-value among successors
        result, f[best] ← RBFS(problem, best, min(f_limit, alternative))
        if result ≠ failure then return result
```

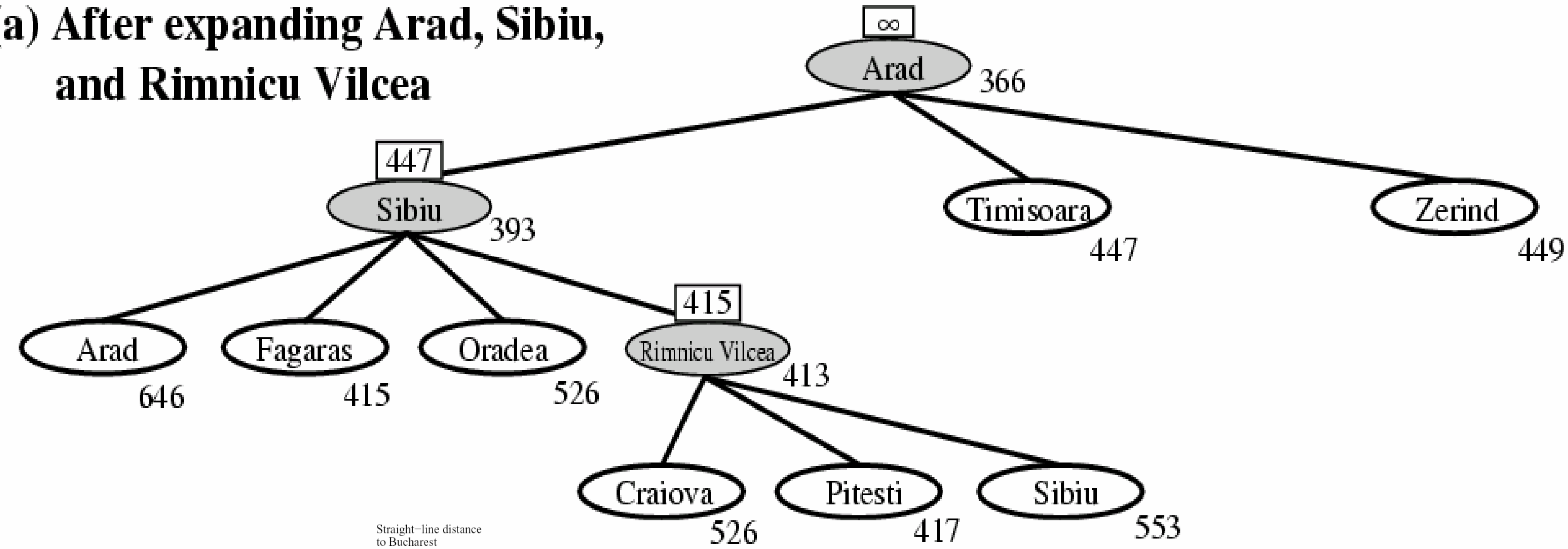
Evaluation function made monotonously increasing ?

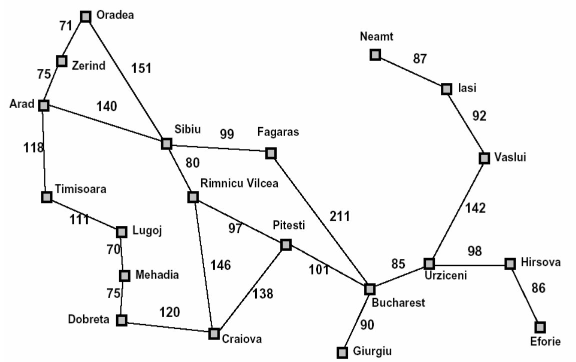
A child node

Recursive Best-First Search (cont.)

- Example: the route-finding problem

(a) After expanding Arad, Sibiu, and Rimnicu Vilcea





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Recursive Best-First Search (cont.)

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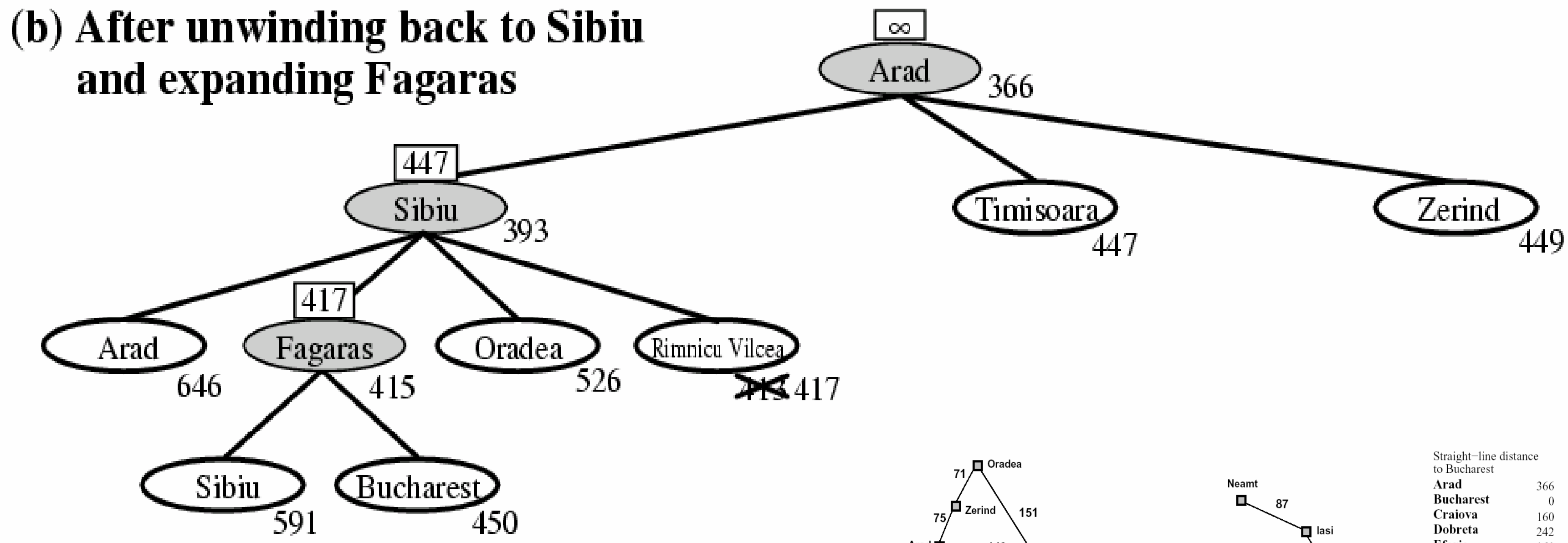
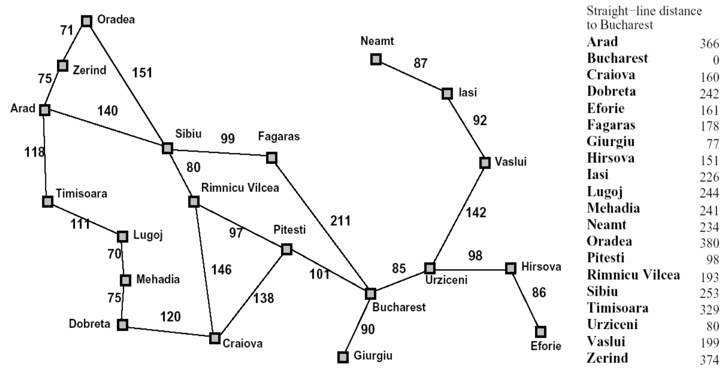
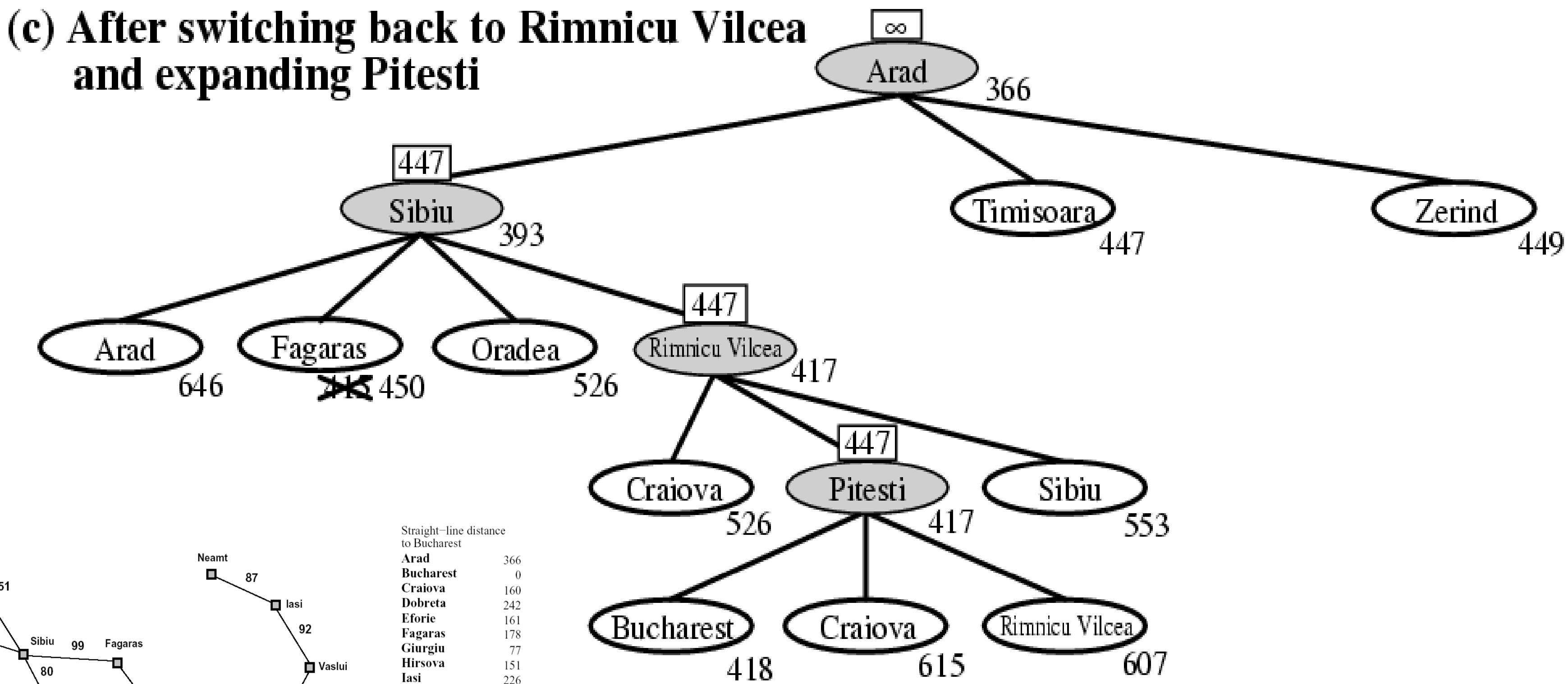
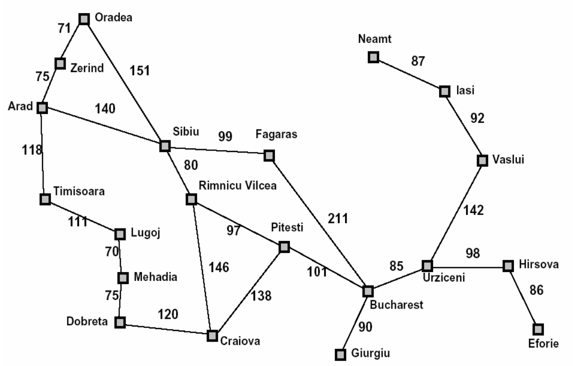




Recursive Best-First Search (cont.)

- Example: the route-finding problem





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Re-expand the forgotten nodes
(subtree of Rimnicu Vilcea)

Properties of RBFS

- RBFS is complete and optimal
- Space complexity: $O(bd)$
- Time complexity : worse case $O(b^d)$
  - Depend on the heuristics and frequency of "mind change"
  - The same states may be explored many times

Simplified Memory-Bounded A* Search (SMA*)

- Make use of all available memory *M* to carry out A*
- Expanding the best leaf like A* until memory is full
- When full, drop the worst leaf node (with highest f -value)
  - Like RBFS, backup the value of the forgotten node to its parent if it is the best among the subtree of its parent
  - When all children nodes were deleted/dropped, put the parent node to the fringe again for further expansion

Simplified Memory-Bounded A* Search (cont.)

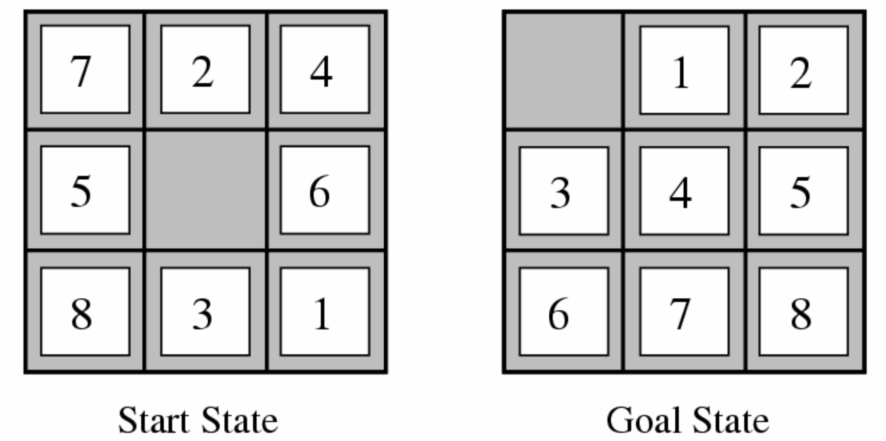
```
function SMA*(problem) returns a solution sequence
   inputs: problem, a problem
   static: Queue, a queue of nodes ordered by f-cost

   Queue ← MAKE-QUEUE({MAKE-NODE(INITIAL-STATE[problem])})
   loop do
       if Queue is empty then return failure
       n ← deepest least-f-cost node in Queue
       if GOAL-TEST(n) then return success
       s ← NEXT-SUCCESSOR(n)
       if s is not a goal and is at maximum depth then
           f(s) ← ∞
       else
           f(s) ← MAX(f(n), g(s)+h(s))
       if all of n's successors have been generated then
           update n's f-cost and those of its ancestors if necessary
       if SUCCESSORS(n) all in memory then remove n from Queue
       if memory is full then
           delete shallowest, highest-f-cost node in Queue
           remove it from its parent's successor list
           insert its parent on Queue if necessary
       insert s on Queue
   end
```

Properties of SMA*

- Is complete if $M \geq d$
- Is optimal if M $\geq d$
- Space complexity: $O(M)$
- Time complexity : worse case $O(b^d)$

Admissible Heuristics

- Take the 8-puzzle problem for example
  - Two heuristic functions considered here
    - $h_1(n)$: number of misplaced tiles
    - $h_2(n)$: the sum of the distances of the tiles from their goal positions (tiles can move vertically, horizontally), also called Manhattan distance or city block distance



- $h_1(n)$: 8
- $h_2(n)$: 3+1+2+2+2+3+3+2=18

Admissible Heuristics (cont.)

- Take the 8-puzzle problem for example
  - Comparison of IDS and A*

branching factor for
8-puzzle: 2~4

solution
length

	Search Cost			Effective Branching Factor		
d	IDS	$A^*(h_1)$	$A^*(h_2)$	IDS	$A^*(h_1)$	$A^*(h_2)$
2	10	6	6	2.45	1.79	1.79
4	112	13	12	2.87	1.48	1.45
6	680	20	18	2.73	1.34	1.30
8	6384	39	25	2.80	1.33	1.24
10	47127	93	39	2.79	1.38	1.22
12	364404	227	73	2.78	1.42	1.24
14	3473941	539	113	2.83	1.44	1.23
16	–	1301	211	–	1.45	1.25
18	–	3056	363	–	1.46	1.26
20	–	7276	676	–	1.47	1.27
22	–	18094	1219	–	1.48	1.28
24	–	39135	1641	–	1.48	1.26

Figure 4.8 Comparison of the search costs and effective branching factors for the ITERATIVE-DEEPENING-SEARCH and A* algorithms with h_1, h_2. Data are averaged over 100 instances of the 8-puzzle, for various solution lengths.

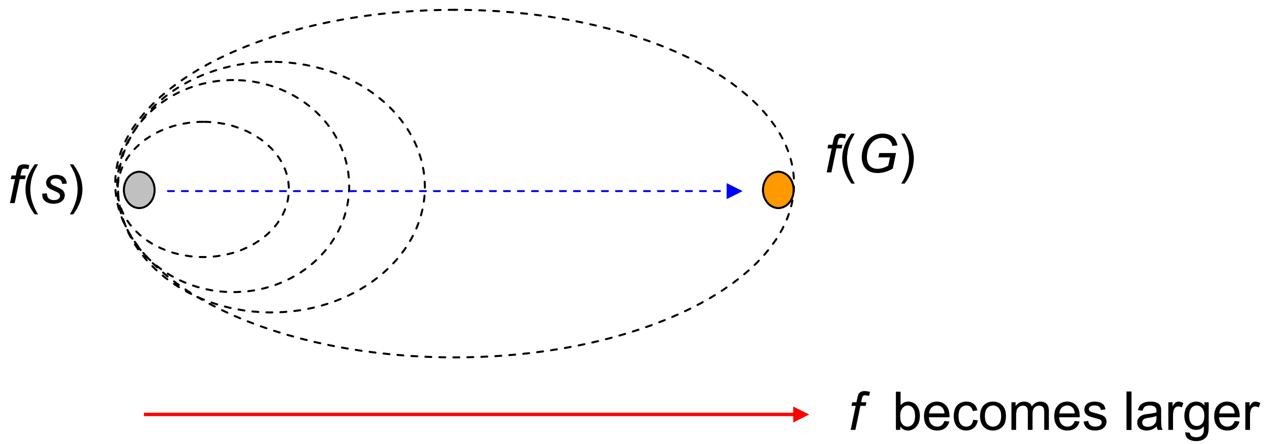
100 random problems
for each number

$$\underbrace{N+1}_{\text{Nodes generated by A*}}=1+ b^*+(b^*)^2+(b^*)^3+\ldots +(b^*)^d$$

*b**: effective branching factor

Dominance

- For two heuristic functions h_1 and h_2 (both are admissible), if $h_2(n) \geq h_1(n)$ for all nodes n
  - Then h_2 **dominates** h_1 and is better for search
  - A* using h_2 will not expand more node than A* using h_1

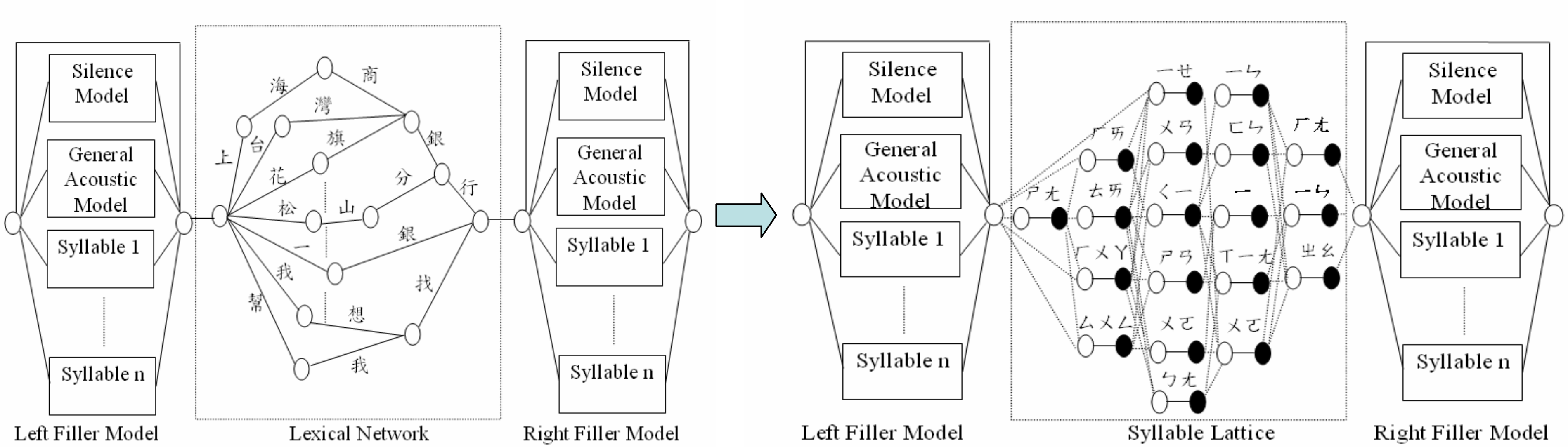


Inventing Admissible Heuristics

- Relaxed Problems
  - The search heuristics can be achieved from the relaxed versions the original problem
    - Key point: the optimal solution cost to a relaxed problem is an admissible heuristic for the original problem
      (not greater than the optimal solution cost of the original problem)
  - Example 1: the 8-puzzle problem
    - If the rules are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the shortest solution
    - If the rules are relaxed so that a tile can move any adjacent square, then $h_2(n)$ gives the shortest solution

Inventing Admissible Heuristics (cont.)

– Example 2: the speech recognition problem



Original Problem
(keyword spotting)

Relaxed Problem
(used for heuristic calculation)

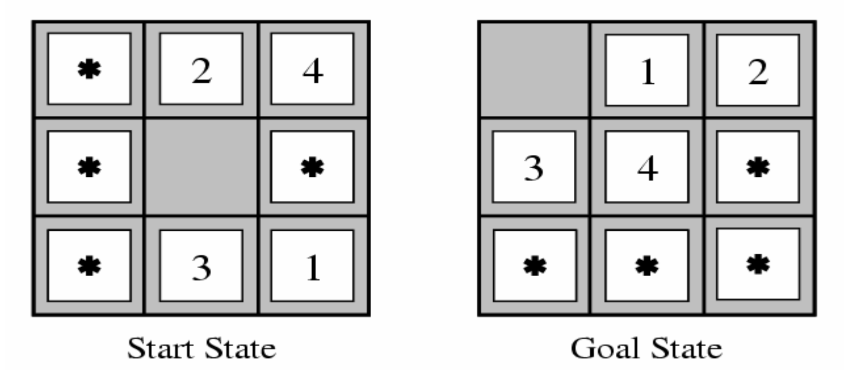
Note: if the relaxed problem is hard to solve, then the values of the corresponding heuristic will be expansive to obtain

Inventing Admissible Heuristics (cont.)

- Composite Heuristics
  - Given a collection of admissible heuristics $h_1, h_2, \ldots, h_m$, none of them dominates any of others

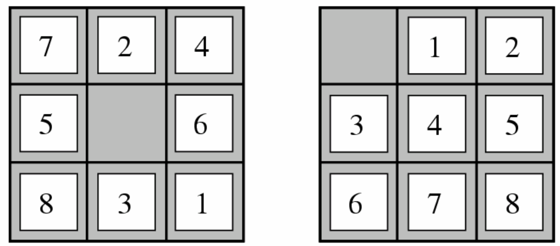
$$h(n) = \max\{h_1(n), h_2(n), \ldots, h_m(n)\}$$

- Subproblem Heuristics
  - The cost of the optimal solution of the subproblem is a lower bound on the cost of the complete problem



Inventing Admissible Heuristics (cont.)

- Inductive Learning
  - E.g., the 8-puzzle problem



Start State Goal State

$x_a(n)$	$x_a(n)$	$h'(n)$
5	4	14
3	6	11
6	3	16
.	.	.
.	.	.
2	7	9

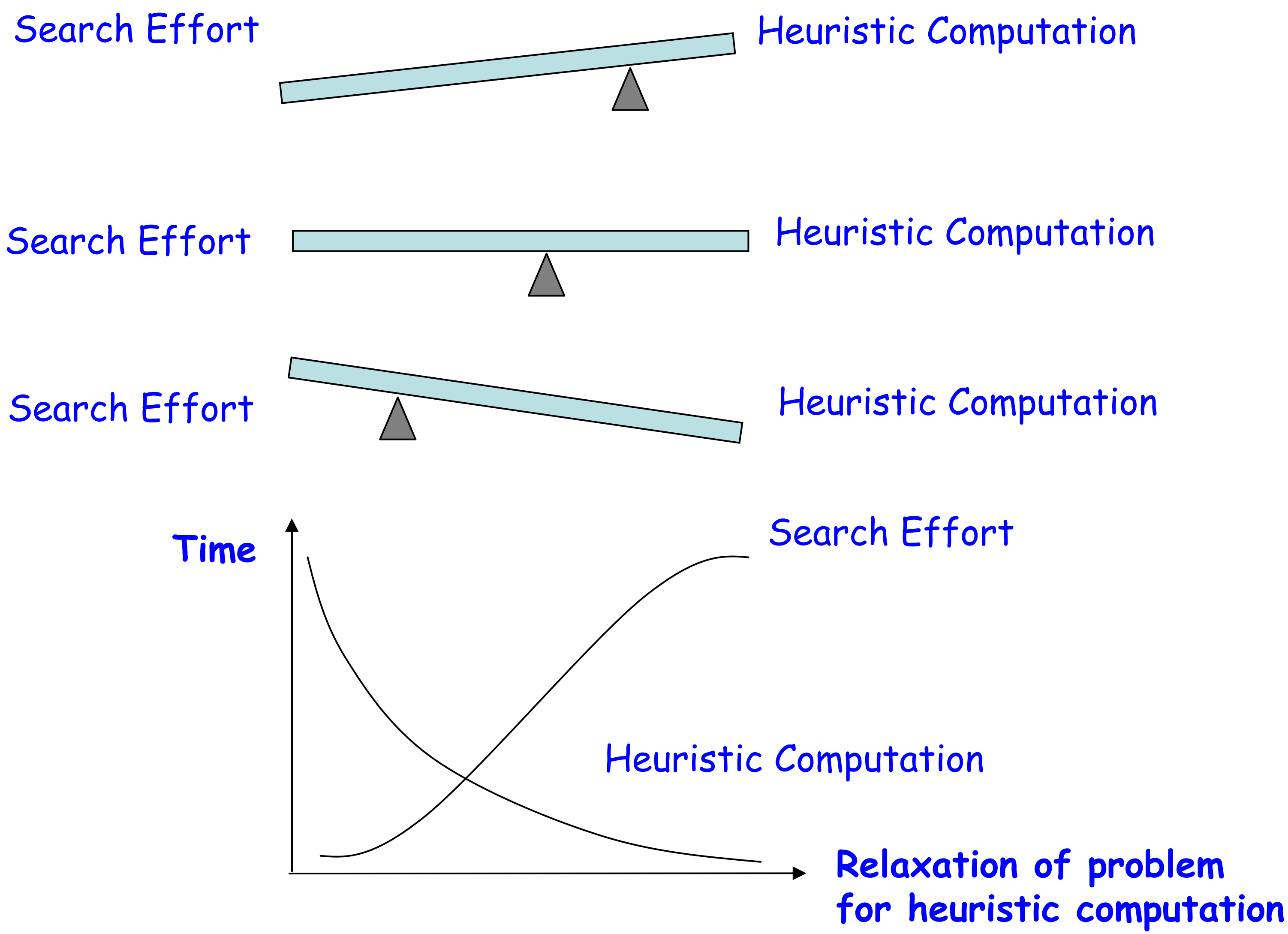
$x_a(n)$: number of misplaced tiles
$x_b(n)$: number of pairs of adjacent tiles that are adjacent in the goal state

$$h'(n) = C_a \cdot x_a(n) + C_b \cdot x_b(n)$$

Linear combination

C_a =? C_b =?

Tradeoffs


Search Effort
Heuristic Computation
Search Effort
Heuristic Computation
Search Effort
Heuristic Computation
Time
Search Effort
Heuristic Computation
Relaxation of problem
for heuristic computation

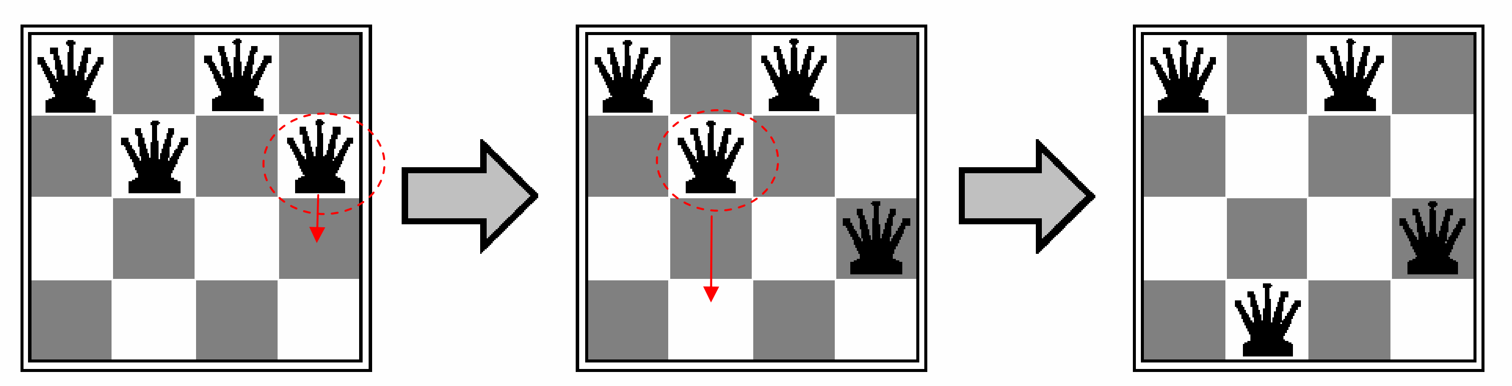
Iterative Improvement Algorithms

- In many optimization, path to solution is irrelevant
  - E.g., 8-queen, VLSI layout, TSP etc., for finding optimal configuration
  - The goal state itself is the solution
  - The state space is a complete configuration

- In such case, iterative improvement algorithms can be used
  - Start with a complete configuration (represented by a single "current" state)
  - Make modifications to improve the quality

Iterative Improvement Algorithms (cont.)

- Example: the *n*-queens problem
  - Put *n* queens on an *nxn* board with no queens on the same row, column, or diagonal
  - Move a queen to reduce number of conflicts

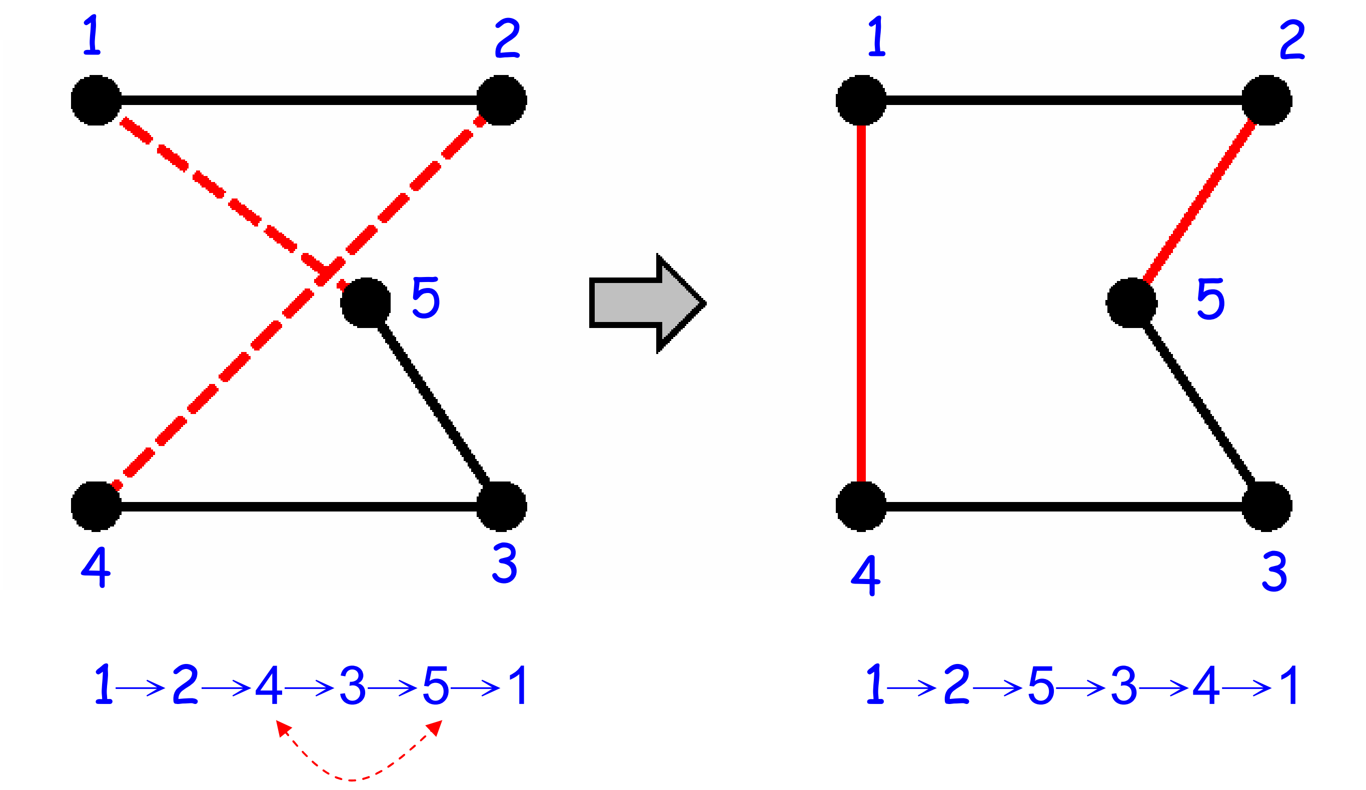


(4, 3, 4, 3) (4, 3, 4, 2) (4, 1, 4, 2)

5 conflicts 3 conflicts 1 conflict

Iterative Improvement Algorithms (cont.)

- Example: the traveling salesperson problem (TSP)
  - Find the shortest tour visiting all cities exactly one
  - Start with any complete tour, perform pairwise exchanges

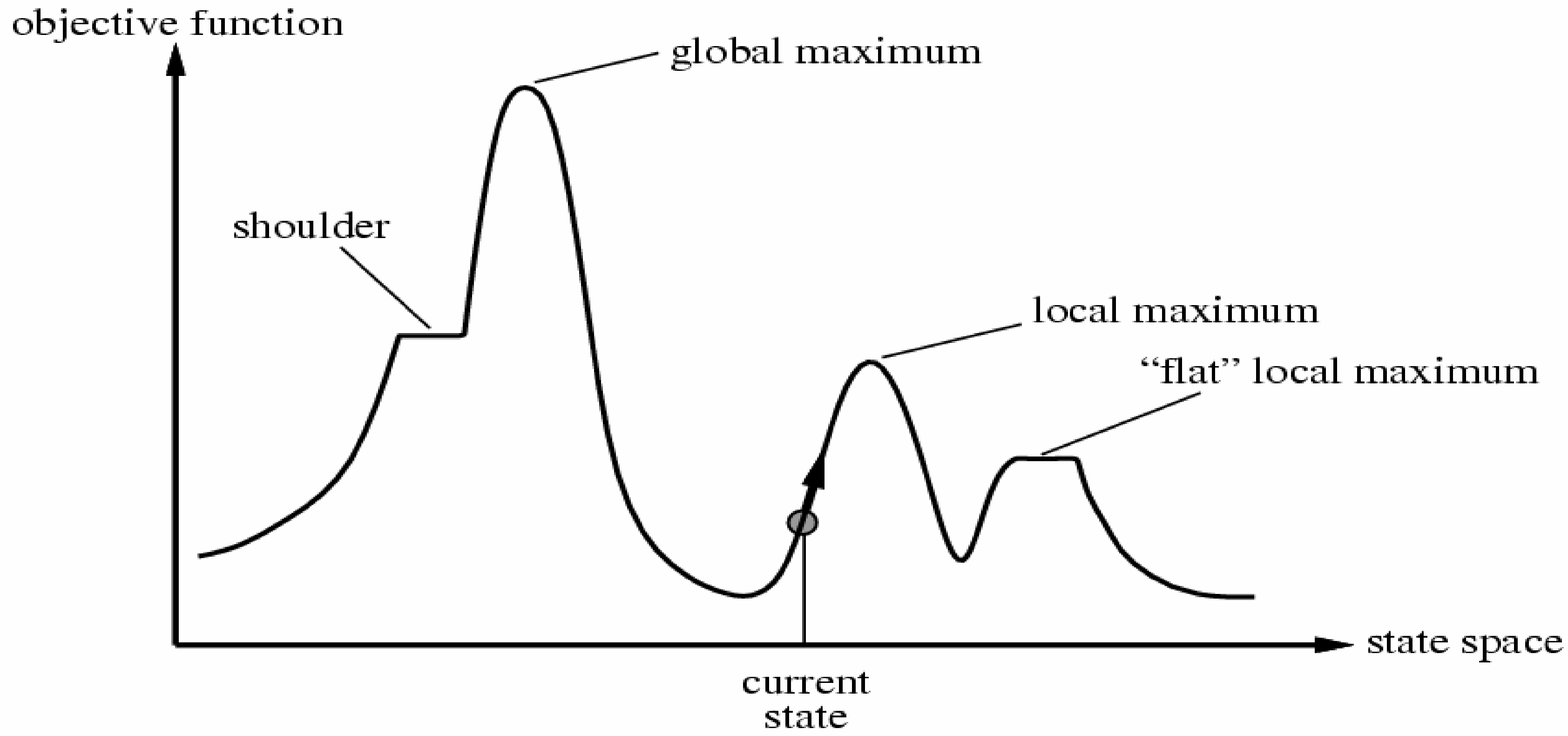


Iterative Improvement Algorithms (cont.)

- *Local search algorithms* belongs to iterative improvement algorithms
  - Use a current state and generally move only to the neighbors of that state
  - Properties
    - Use very little memory
    - Applicable to problems with large or infinite state space

- Local search algorithms to be considered
  - Hill-climbing search
  - Simulated annealing
  - Local beam search
  - Genetic algorithms

Iterative Improvement Algorithms (cont.)



- *Completeness* or *optimality* of the local search algorithms should be considered

Hill-Climbing Search

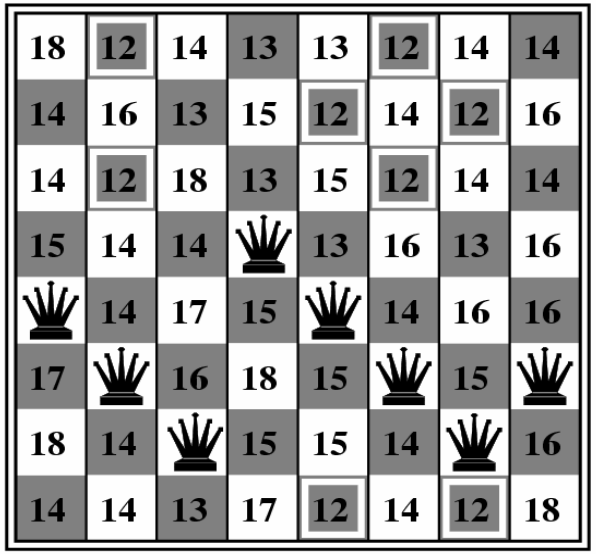
- "Like climbing Everest in the thick fog with amnesia"
- Choose any successor with a higher value (of objective or heuristic functions) than current state
  - Choose Value[next] $\geq$ Value[current]

```
function HILL-CLIMBING(problem) returns a state that is a local maximum
  inputs: problem, a problem
  local variables: current, a node
                   neighbor, a node

  current ← MAKE-NODE(INITIAL-STATE[problem])
  loop do
     neighbor ← a highest-valued successor of current
     if VALUE[neighbor] ≤ VALUE[current] then return STATE[current]
     current ← neighbor
```

- Also called *greedy local search*

Hill-Climbing Search (cont.)

- Example: the 8-queens problem
  – The heuristic cost function is the number of pairs of queens that are attacking each other



  – h=3+4+2+3+2+2+1=17 (calculated from left to right)
  – Best successors have h=12
    (when one of queens in Column 2,5,6, and 7 is moved)

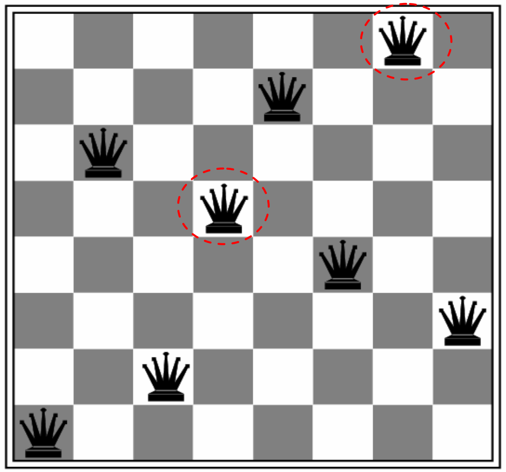
Hill-Climbing Search (cont.)

- Problems:
  - Local maxima: search halts prematurely
  - Plateaus: search conducts a random walk
  - Ridges: search oscillates with slow progress (resulting in a set of maxima)

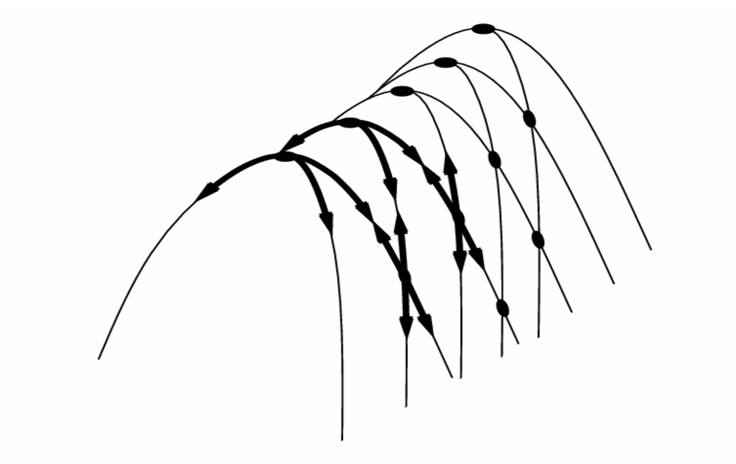
Neither complete nor optimal

- Solution ?

8-queens stuck in a local minimum



Ridges cause oscillation



Hill-Climbing Search (cont.)

- Several variants
  - Stochastic hill climbing
    - Choose at random from among the uphill moves
  - First-choice hill climbing
    - Generate successors randomly until one that is better than current state is generated
    - A kind of stochastic hill climbing
  - Random-restart hill climbing
    - Conduct a series of hill-climbing searches from randomly generated initial states
    - Stop when goal is found

Simulated Annealing Search

- Combine hill climbing with a random walk to yield both efficiency and completeness
  - Pick a random move at each iteration instead of picking the best move
  - If the move improve the situation $\rightarrow$ accept!

$$\Delta E = \text{VALUE}[next] - \text{VALUE}[current]$$

  - Otherwise($\Delta E < 0$) , have a probability ($e^{\Delta E / T}$) to move to a worse state
    - The probability decreases exponentially as ΔE decreases
    - The probability decreases exponentially as T (temperature) goes down (as time goes by)

Simulated Annealing Search (cont.)

```
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
   inputs: problem, a problem
           schedule, a mapping from time to "temperature"
   local variables: current, a node
                    next, a node
                    T, a "temperature" controlling the probability of downward steps

   current ← MAKE-NODE(INITIAL-STATE[problem])
   for t ← 1 to ∞ do
       T ← schedule[t]
       if T = 0 then return current
       next ← a randomly selected successor of current
       ΔE ← VALUE[next] – VALUE[current]
       if ΔE > 0 then current ← next
       else current ← next only with probability e^(ΔE/T)
```

Be negative here!

Local Beam Search

- Keep track of *k* states rather than just one
  - Begin with *k* randomly generated states
  - All successors of the *k* states are generated at each iteration
    - If any one is a goal → halt!
    - Otherwise, select *k* best successors from them and continue the iteration
  - Information is passed/exchanged among these *k* search threads
    - Compared to the random-restart search
      - Each process run independently

Local Beam Search (cont.)

- Problem
  - The k states may quickly become concentrated in a small region of the state space
  - Like an expensive version of hill climbing

- Solution
  - A variant version called **stochastic beam search**
    - Choose a given successor at random with a probability in increasing function of its value
    - Resemble the process of natural selection

Genetic Algorithms (GAs)

- Developed and patterned after biological evolution

- Also regarded as a variant of stochastic beam search
  - Successors are generated from multiple current states
    - A population of potential solutions are maintained
  - States are often described by bit strings (like chromosomes) whose interpretation depends on the applications
    - Binary-coded or alphabet

      (11, 6, 9) $\rightarrow$ (101101101001)
    - Encoding: translate problem-specific knowledge to GA framework
  - Search begins with a population of randomly generated initial states

Genetic Algorithms (cont.)

- The successor states are generated by combining two parent states, rather then by modifying a single state
  - Current population/states are evaluated with a *fitness function* and selected probabilistically as seeds for producing the next generation
    - Fitness function: the criteria for ranking
    - Recombine parts of the best (most fit) currently known states
    - Generate-and-test beam search

- Three phases of GAs
  - Selection → Crossover → Mutation

Genetic Algorithms (cont.)

- Selection
  - Determine which parent strings (chromosomes) participate in producing offspring for the next generation
  - The selection probability is proportional to the fitness values

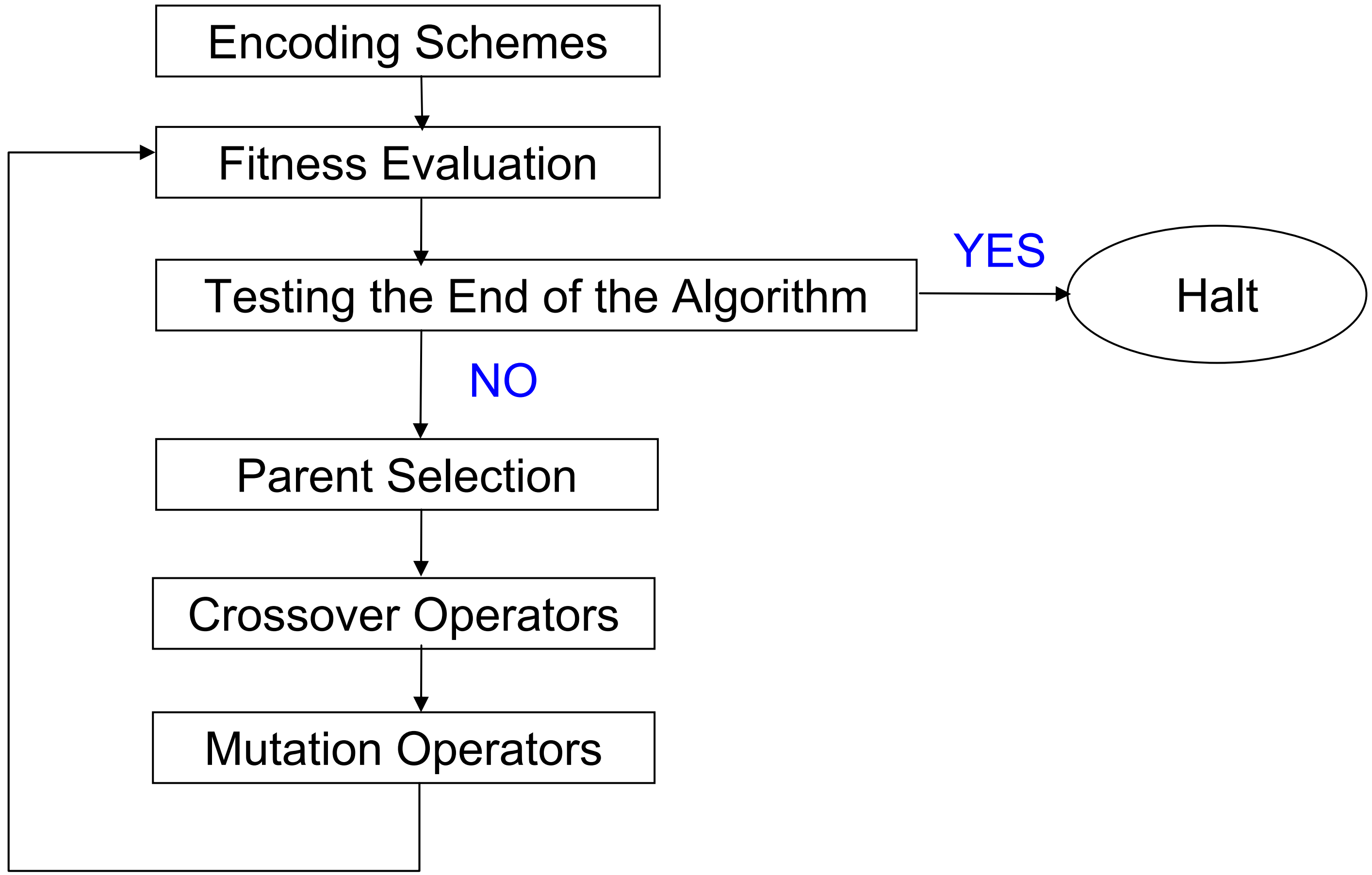
$$\Pr(h_i) = \frac{Fitness\,(h_i)}{\sum_{j=1}^{P} Fitness\,(h_j)}$$

  - Some strings (chromosomes) would be selected more than once

Genetic Algorithms (cont.)

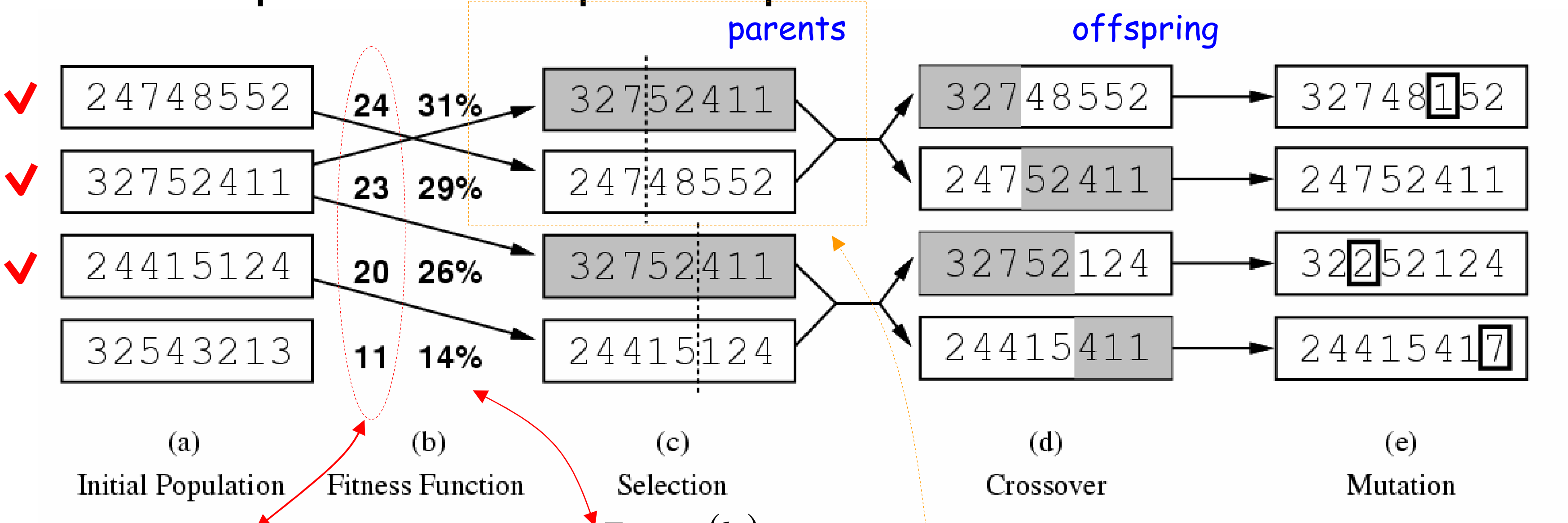
- Two most common (genetic) operators which try to mimic biological evolution are performed at each iteration
  - Crossover
    - Produce new offspring by crossing over the two mated parent strings at randomly (a) chosen crossover point(s) (bit position(s))
    - Selected bits copied from each parent
  - Mutation
    - Often performed after crossover
    - Each (bit) location of the randomly selected offspring is subject to random mutation with a small independent probability
- Applicable problems
  - Function approximation & optimization, circuit layout etc.

Genetic Algorithms (cont.)



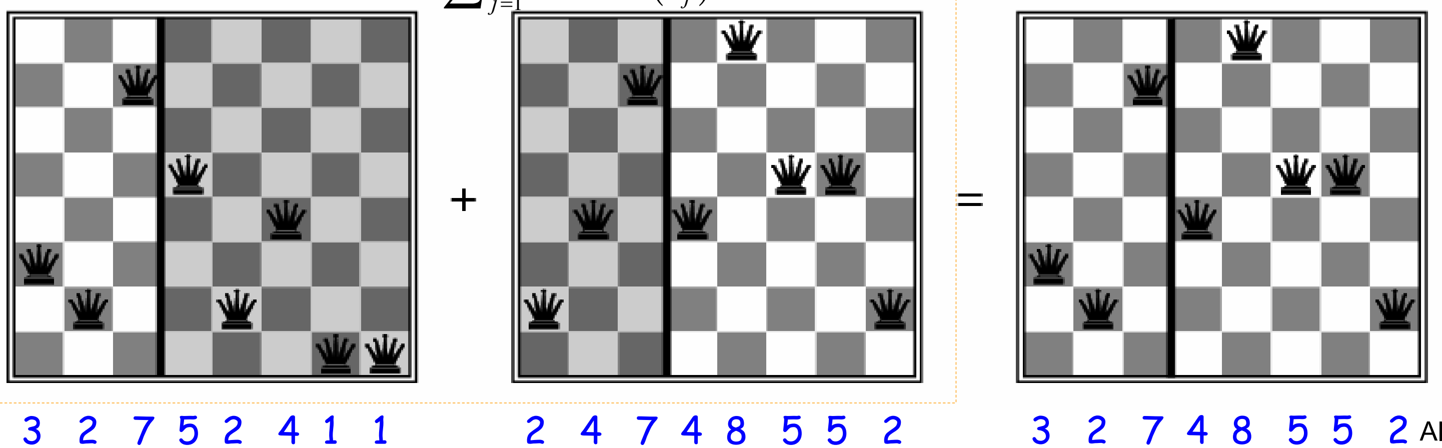
Genetic Algorithms (cont.)

- Example 1: the 8-queens problem



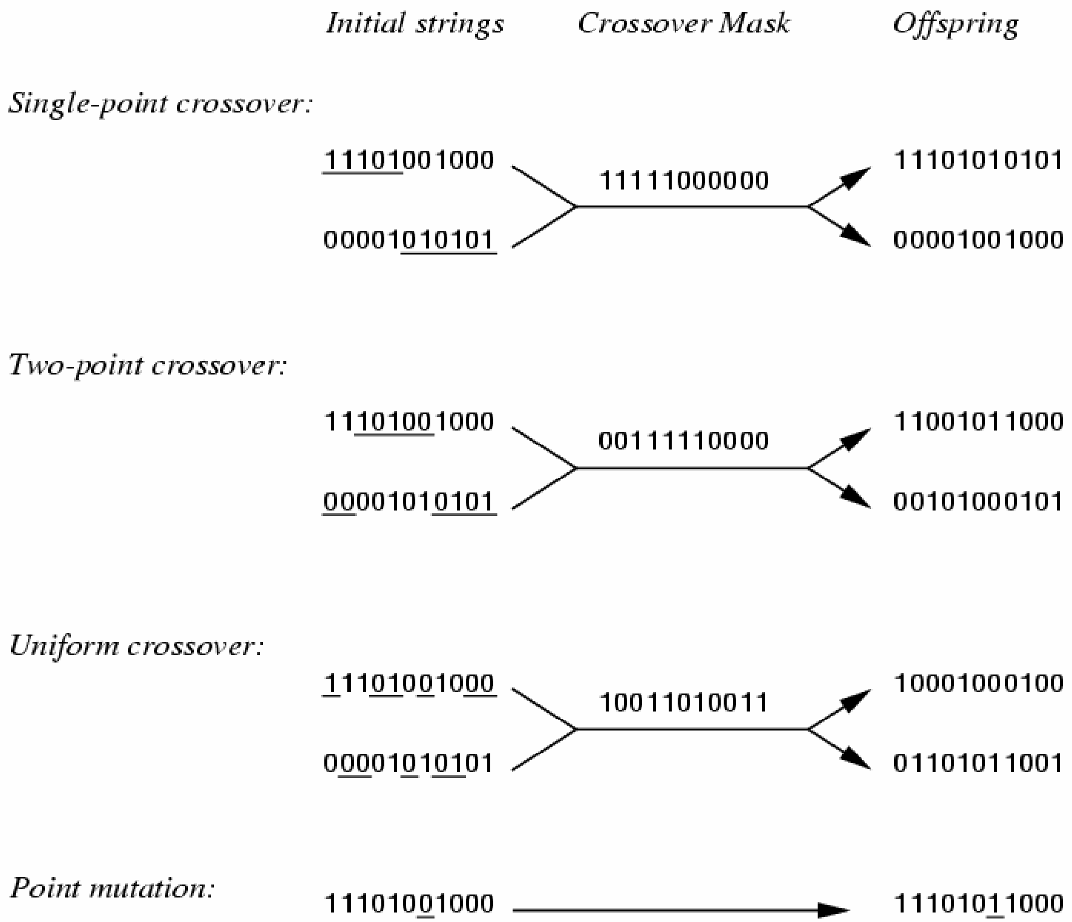
number of non attacking pairs of queens

$$\Pr(h_i) = \frac{Fitness(h_i)}{\sum_{j=1}^{P} Fitness(h_j)}$$



Genetic Algorithms (cont.)

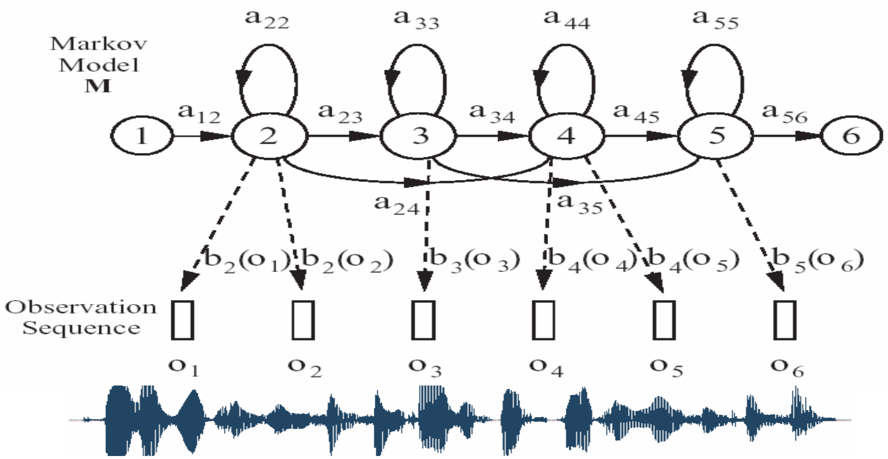
- Example 2: common crossover operators



Genetic Algorithms (cont.)

- Example 3: HMM adaptation in Speech Recognition

$$\Pr(\boldsymbol{h}_i)=\frac{\exp\left(\frac{P(\boldsymbol{O}|\boldsymbol{h}_i)}{T}\right)}{\sum_{j=1}^{P}\exp\left(\frac{P(\boldsymbol{O}|\boldsymbol{h}_j)}{T}\right)}$$



sequences of HMM mean vectors

$$\boldsymbol{h}_1=(k_1,k_2,k_3,....,k_D)$$
$$\boldsymbol{h}_2=(m_1,m_2,m_3,....,m_D)$$

crossover (reproduction)

$$\boldsymbol{s}_1=\left(k_1\cdot i_f+m_1\cdot(1-i_f),k_2\cdot i_f+m_2(1-i_f),m_3\cdot i_f+k_3(1-i_f),....,m_3\cdot i_f+k_D(1-i_f)\right)$$
$$\boldsymbol{s}_2=\left(m_1\cdot i_f+k_1\cdot(1-i_f),m_2\cdot i_f+k_2(1-i_f),k_3\cdot i_f+m_3(1-i_f),....,k_3\cdot i_f+m_D(1-i_f)\right)$$

$$g_d \Longrightarrow \hat{g}_d=g_d+\varepsilon\cdot\sigma_d$$

mutation

Genetic Algorithms (cont.)

```
function GENETIC-ALGORITHM(population, FITNESS-FN) returns an individual
  inputs: population, a set of individuals
          FITNESS-FN, a function that measures the fitness of an individual

  repeat
     new_population ← empty set
     loop for i from 1 to SIZE(population) do
        x ← RANDOM-SELECTION(population, FITNESS-FN)
        y ← RANDOM-SELECTION(population, FITNESS-FN)
        child ← REPRODUCE(x, y)
        if (small random probability) then child ← MUTATE(child)
        add child to new_population
     population ← new_population
  until some individual is fit enough, or enough time has elapsed
  return the best individual in population, according to FITNESS-FN
```

```
function REPRODUCE(x, y) returns an individual
  inputs: x, y, parent individuals

  n ← LENGTH(x)
  c ← random number from 1 to n
  return APPEND(SUBSTRING(x, 1, c), SUBSTRING(y, c + 1, n))
```

Genetic Algorithms (cont.)

- Main issues
  - Encoding schemes
    - Representation of problem states
  - Size of population
    - Too small → converging too quickly, and vice versa
  - Fitness function
    - The objective function for optimization/maximization
    - Ranking members in a population

Properties of GAs

- GAs conduct a randomized, parallel, hill-climbing search for states that optimize a predefined fitness function

- GAs are based an analogy to biological evolution

- It is not clear whether the appeal of GAs arises from their performance or from their aesthetically pleasing origins in the theory of evolution

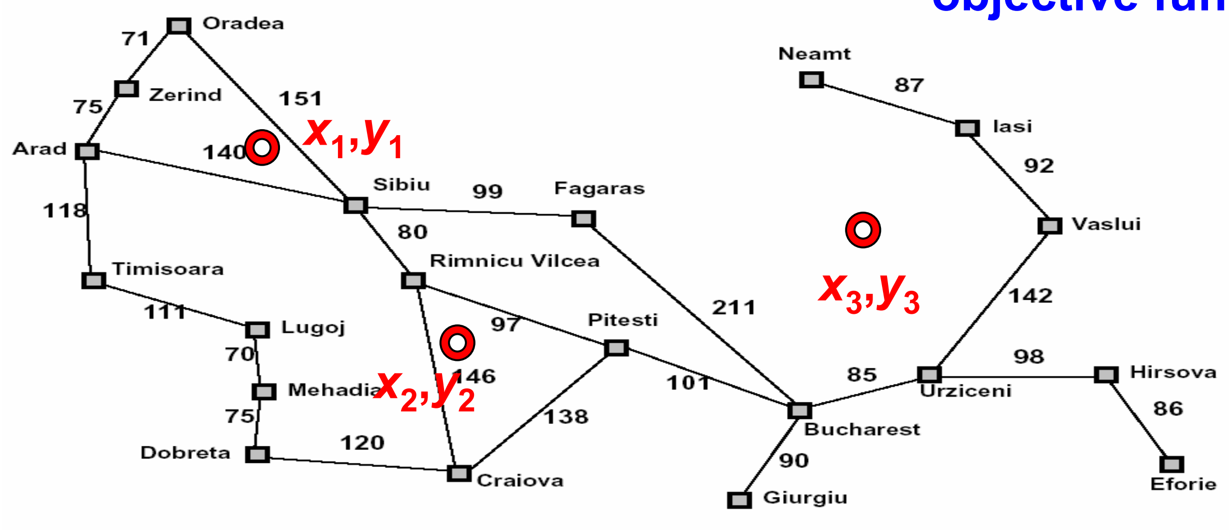
Local Search in Continuous Spaces

- Most real-world environments are continuous
  - The successors of a given state could be infinite

- Example:

  Place three new airports anywhere in Romania, such that the sum of squared distances from each cities to its nearest airport is minimized

  objective function: *f* =?



Local Search in Continuous Spaces (cont.)

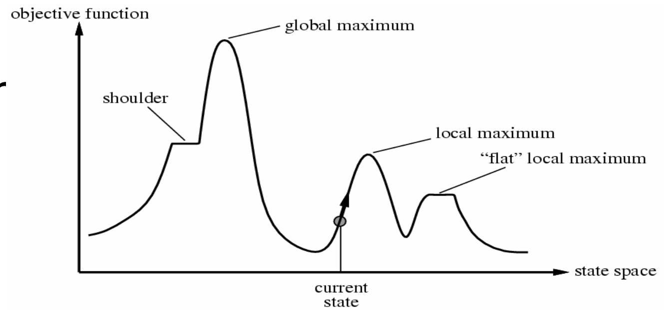
- Two main approach to find the maximum or minimum of the objective function by taking the gradient

  1. Set the gradient to be equal to zero (=0) and try to find the closed form solution
     - If it exists → *lucky!*

  2. If no closed form solution exists
     - Perform gradient search !

Local Search in Continuous Spaces (cont.)

- Gradient Search
  - A hill climbing method
  - Search in the space defined by the real numbers
  - Guaranteed to find local maximum
  - Not Guaranteed to find global maximum



maximization

the gradient of objective function

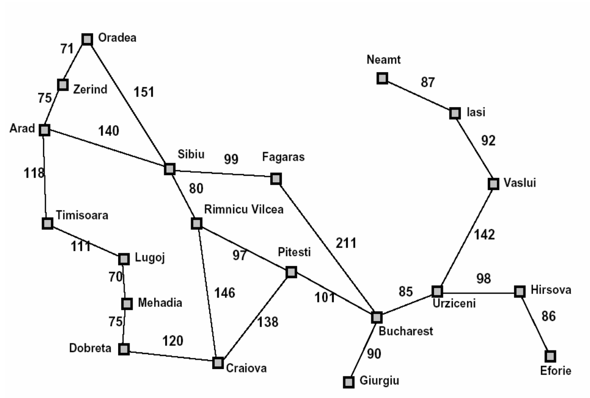
$$\hat{\boldsymbol{x}} = \boldsymbol{x} + \alpha \boxed{\nabla f(\boldsymbol{x})} = \boldsymbol{x} + \alpha \frac{df(\boldsymbol{x})}{d\boldsymbol{x}}$$

minimization

$$\hat{\boldsymbol{x}} = \boldsymbol{x} - \alpha \nabla f(\boldsymbol{x}) = \boldsymbol{x} - \alpha \frac{df(\boldsymbol{x})}{d\boldsymbol{x}}$$

Online Search

- Offline search mentioned previously
  - Nodes expansion involves simulated rather real actions
  - Easy to expand a node in one part of the search space and then immediately expand a node in another part of the search space



- Online search
  - Expand a node physically occupied
    - The next node expanded (except when backtracking) is the child of previous node expanded
  - Traveling all the way across the tree to expand the next node is costly

Online Search (cont.)

- Algorithms for online search
  - Depth-first search
    - If the actions of agent is reversible (backtracking is allowable)
  - Hill-climbing search
    - However random restarts are prohibitive
  - Random walk
    - Select at random one of the available actions from current state
    - Could take exponentially many steps to find the goal