

Linear Algebra

Homework 2

Due date: 2015/10/23

Note: You have to answer the questions with supporting explanations if needed. The computations have to be accomplished with paper and pencil.

1. Find the determinant of the following matrix by a cofactor expansion along a row or a column of your choice.

$$\begin{bmatrix} 3 & 4 & 3 & 0 \\ 5 & 4 & 6 & 6 \\ -1 & 2 & 3 & 0 \\ 4 & 2 & 4 & 3 \end{bmatrix}$$

2. Find the determinant of the following matrix by row reduction.

$$\begin{bmatrix} 1 & 3 & -1 & 0 & -2 \\ 0 & 2 & -4 & -1 & -6 \\ 3 & 7 & -3 & 8 & -7 \\ 3 & 5 & 5 & 2 & 7 \\ -2 & -6 & 2 & 3 & 9 \end{bmatrix}$$

3. Solve the following linear system with the Cramer's rule, if it can be applied.

$$\begin{aligned} 3x_1 + x_2 + 2x_3 &= 5 \\ -x_1 + 4x_2 - 3x_3 &= 0 \\ 3x_1 + 5x_2 &= 7 \end{aligned}$$

4. Given that A is a 5×5 matrix for which $\det(A) = -2$. Solve the following expressions.

$$(i) \det(5A) \quad (ii) \det(A^{-1}) \quad (iii) \det(-2(A^{-1})^2) \quad (iv) \det((2A)^{-1})$$

5. Given that B is a $n \times n$ matrix, prove that:

$$\det(\text{adj}(B)) = \det(B^{n-1})$$